East (see Courseware News for more details). The basic DIAGNOSYS system v2.2 is now available in the subdirectory `tltp/diagnosy`. Materials from Mathwise and the Maths Assessor are available in the directories `tltp/mathwise` and `assessor`, and there are pointers to other material. The server address is `ccbl.bham.ac.uk`, login ctimath, password your own email address.

World Wide Web: CTI Mathematics has a Web server at URL `http://www.bham.ac.uk/ctimath/`, with copies of the files from the original text-based server cti-server@bham.ac.uk, and pointers to other mathematics resources. A full list of information sheets available in printed or electronic form can be found under the heading General Information and cover the following topics: the Centre and its organisation, workshop proceedings, TLTP and related CTI projects and software reviews. We intend to publish illustrated articles and reviews in future, including electronic extracts from the newsletter. CTI Statistics has its own separate Web server at URL `http://www.stats.gla.ac.uk:80/cti/`

NISS (National Information Services and Systems) has recently set up an information gateway to provide easy access to many Internet resources, under the headings of news and current affairs, subject and academic disciplines, works of reference and bibliographic services, higher education, administrative and professional, and IT and computing. This last heading contains information on all the CHEST software, training, datasets and hardware deals, the CHEST Directory and numerous other catalogues of IT products including those of HENSA. The address of the NISS gateway is on the Web at `http://www.niss.ac.uk/`
or telnet to `niss.ac.uk` and select option 1
or PAD to `uk.ac.niss` and select option 1

Electronic discussion groups: There are several electronic discussion lists for our community, which provide a forum for exchanging information on general topics of interest. You can join any list by sending an email message to mailbase@mailbase.ac.uk with the single line join <listname> <your name> (for example, join cti-maths Pam Bishop) as the text of the message. Mail sent to the address <listname>@mailbase.ac.uk is automatically forwarded to all members of that list. The lists run by the Centre include *cti-maths,* open to all mathematicians in the UK (and overseas) who want to share information of general interest, and *allstat,* its equivalent for statistics.

Information on Software: At the Centre we maintain a database of software used to support teaching in higher education. To date there are over 1220 entries. Information on any of these packages, or listings under various categories, are available from the Centre on request. The last printed edition of the joint Software Guide for mathematics and statistics is due to be circulated to Heads of Departments in June. Future editions will be published in electronic format, and will be separate in respect of the two disciplines. Look out for full details in this newsletter.

TI-92, the hand-held revolution in computer enhanced maths teaching and learning

Professors Bert K Waits and Franklin Demana, Department of Mathematics, Ohio State University, email: waitsb@math.ohio-state.edu

Inexpensive hand held "super" calculators with built-in graphing software for enhancing mathematics teaching and learning are now 10 years old. Casio invented the first graphing calculator in 1985 and started a revolution in delivering powerful, computer aided graphing to now millions of mathematics students worldwide. Casio was soon joined by Hewlett Packard, Sharp, and Texas Instruments in marketing graphing calculators.

Inexpensive graphing calculators fulfilled our dream of making computer visualization for both in-class and out-of-class activities practical for all students. Expensive computers and software located in expensive to operate (and maintain) computer labs were no longer barriers to regular student use of computer visualization and numerical methods in their mathematics education.

A new generation of hand-held computer tools

The new Texas Instruments TI-92 is the first example of another and much more powerful generation of super calculators. In fact, it is difficult not to call the TI-92 a true computer! It is small, inexpensive ($250 US suggested retail price), hand-held, user-friendly and yet has a large screen display (240 by 128 pixels), QWERTY keyboard, and, most importantly, contains

a collection of very useful and powerful integrated software packages for doing algebra, geometry, calculus, statistics, and matrix algebra.

The built-in software on the TI-92 includes five graphing packages (including an improved sequence grapher and a wonderful 3D grapher), computer interactive geometry co-developed with the authors of CABRI II® Geometry, and a powerful computer algebra system co-developed with the authors of DERIVE® as well as advanced programming software. Computer PC or MAC versions of the built-in software on the TI-92 could cost four or five times the cost of the TI-92.

Remarkably, the TI-92 computer is almost as easy to use as a TI-82 graphing calculator. The TI-82 functionality on the TI-92 is immediately apparent by looking at the familiar TI-82 keys (HOME, Y=, WINDOW, GRAPH, TABLE, etc.) above the QWERTY keys in green. Other features of the TI-92 include exact real and complex arithmetic *and* exact algebraic results printed on the screen like you would see it on the chalk board in class, powerful solvers, user defined functions, various split screen modes, and a powerful text editor with scripting.

This article will focus on some new features of the TI-92 that can be used to enhance the teaching and learning of mathematics. First we show the power of the TI-92 computer algebra system. Figure 1 shows several simple TI-92 computer algebra commands like **factor, expand, and solve** applied to typical algebra and calculus problems. What is remarkable is the speed and ease of obtaining satisfactory CAS results in such an inexpensive, hand-held device.
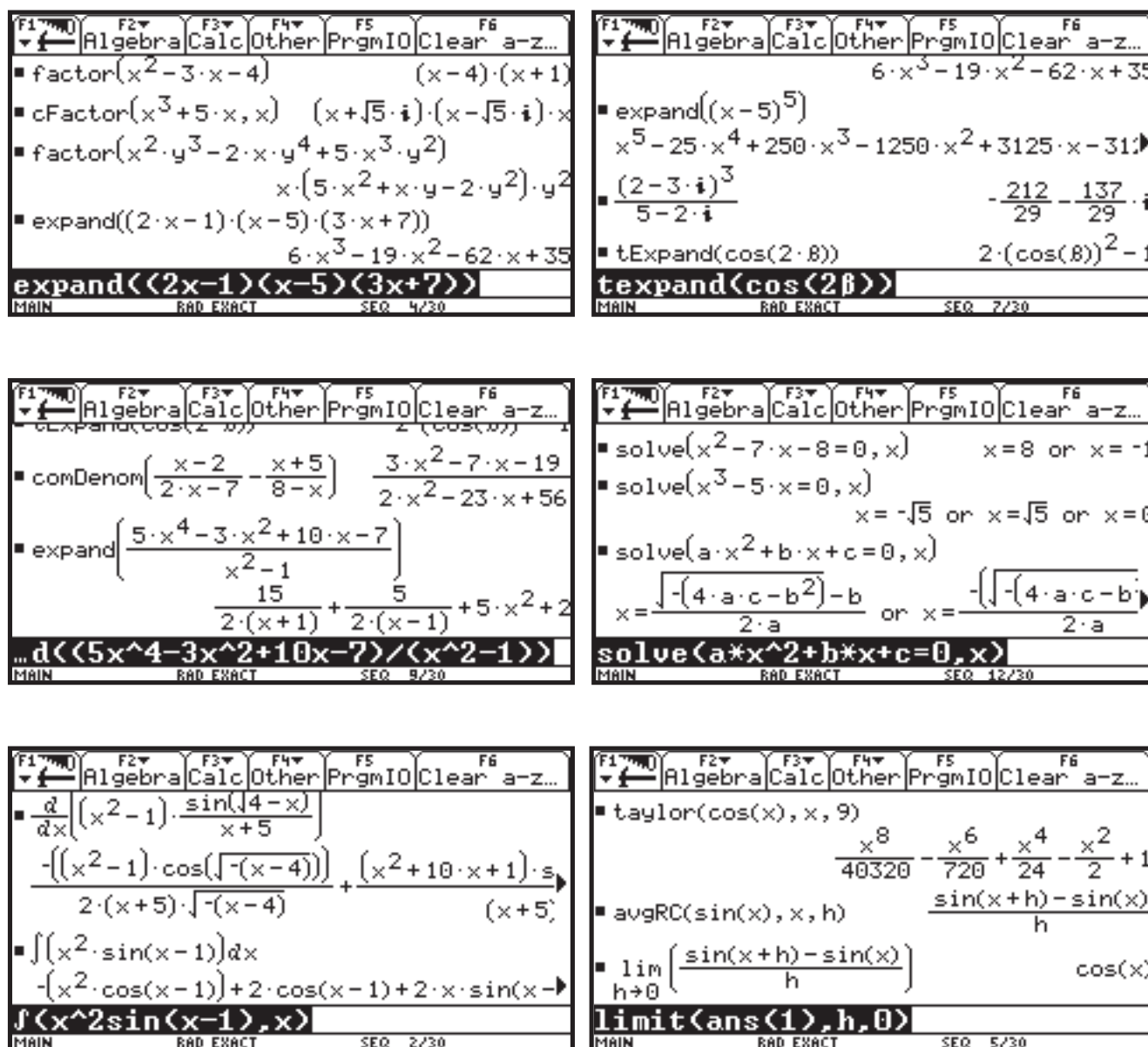

Figure 1. Computer algebra in action on the TI-92

What is more important - concepts or computational methods?

Our first example illustrates that the *method* used to do an algebraic computation (paper and pencil or technology) is not really important. Rather, it is the *concept* that can be illustrated by applying the algorithm or technology that is important. In such cases do we really care if the computations are performed with paper and pencil or computer algebra?

In Figure 2 the TI-92 command "expand" is used to decompose the given rational function into two parts; $20/(x-2)$ and x^2-8x-15. The significance of this decomposition is clear from examining the graphs shown in Figure 2.
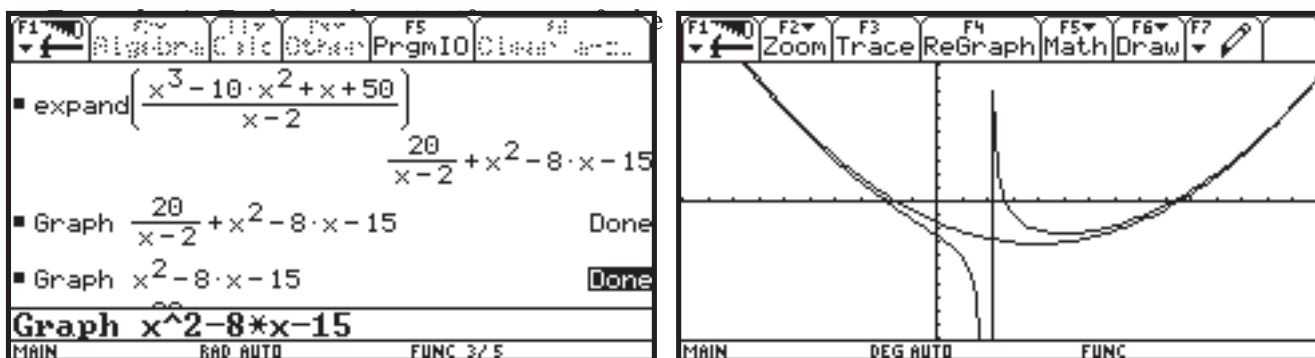
Figure 2. Decomposing a rational function

The graph of the quotient x^2-8x-15 "exactly" describes the behavior of the rational function for values of x "away from 2" while the graph of $20/(x-2)$ describes the behavior for values of x near x=2. The quadratic function $f(x)=x^2$-8x-15 is called the quadratic asymptote of this rational function and is unique. Thus the behavior of a complicated function in its original form can be easily understood by considering its decomposition into "primitive" parts. Again, the tool or algorithm used to obtain the decomposition is not the important consideration. Note the use of the TI-92 "graph" command in the edit line.

Adding excitement and interest to mathematics learning

New graphing modes on the TI-92 have independent "visual" attributes (such as DOT, PATH, ANIMATE,

THICK, etc) for each expression in the Y= editor. Here is the well known "baseball problem" solved using the parametric grapher on the TI-92.

Example 2: A baseball is hit at an initial speed of 95 ft/sec at an initial angle of 25° with the horizontal from ground level. How far does it travel? When does the ball hit the ground? How high does the ball go? We ignore all forces except gravity, etc.

The solution presented here must be done "live" by the student to be appreciated. The split screen displays in Figure 3 outlines the setup of this problem (note: the complete equation for yt1 is given by yt1=95*t*sin(25°) - 16t^2). "Animate" is selected for the (xt1, yt1) pair while "Path" is selected for the other two pairs using the F6 Style menu.
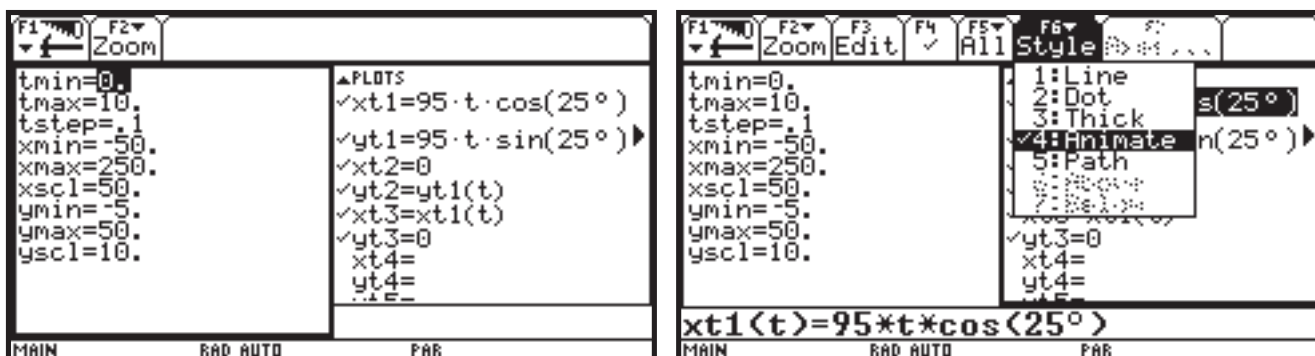

Figure 3. The baseball problem setup

The plotting action shown in Figure 4 shows the ball in flight and, *simultaneously*, the vertical and horizontal components (vectors) of the motion. It is a dramatic visualization that must be done "live" to really be appreciated.
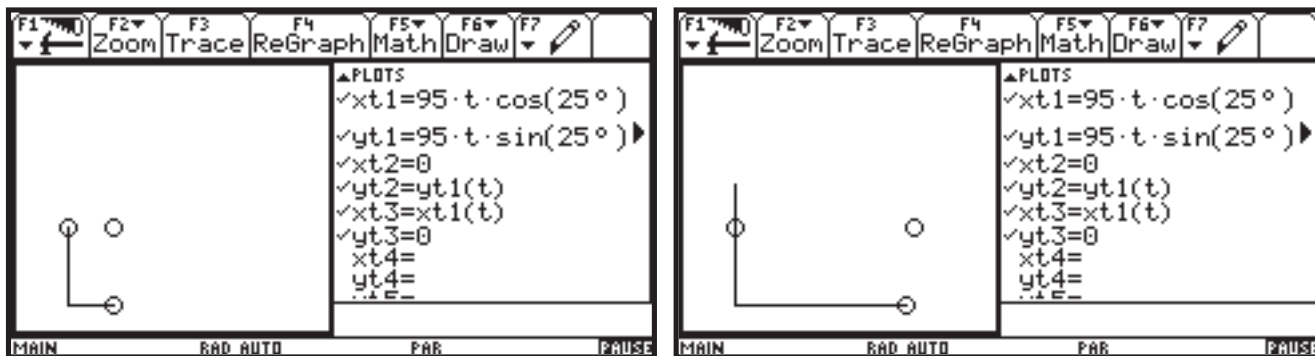

Figure 4. Decomposing the motion of the baseball

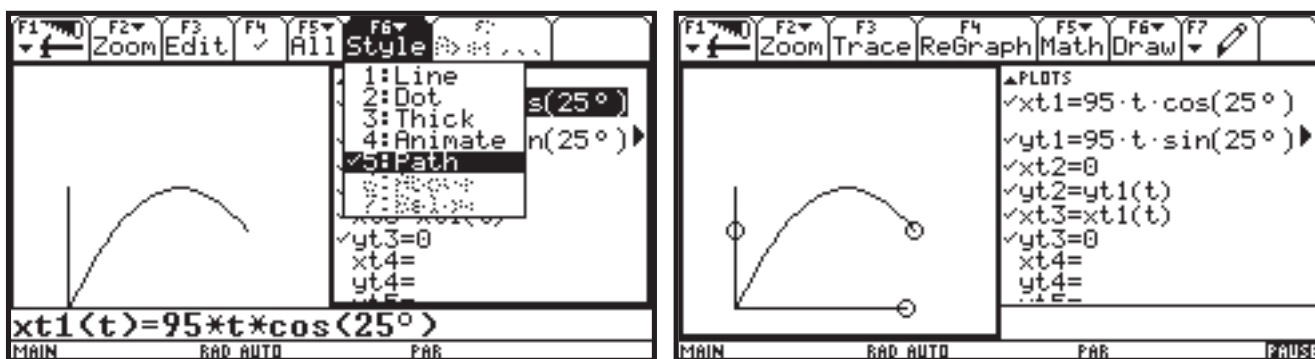In Figure 5, the "Animate" style of the pair (xt1, yt1) is changed to "Path".

Figure 5. Animating the baseball problem

Of course, TRACE can be used to obtain numerical results for the questions in Example 2 as on most graphing calculators. Paper and pencil or computer algebra can be used to find the exact solutions confirming the numerical TRACE results. For example, using the TI-92 CAS, the exact maximum height of the ball in flight is

$$yt1(\text{solve}(d(yt1(t),t)=0,t)) = \frac{9025(\sin(25^o))^2}{64}$$

Geometry and algebra connections!

The next example outlines a wonderful mathematical connection that can be illustrated with the TI-92. We use CABRI geometry, analytic geometry, data analysis, function graphing, and calculus ideas.

Example 3: Discover the function determined by the area "under" the graph of the line y = x bounded by the lines x = 0, x = a, and the x-axis. Generalize.

We start with CABRI Geometry from the APPS menu and select the rectangular coordinate axes option. We use built-in tools to draw an angle bisector through the origin, a perpendicular line to the x-axis, and then a *right* triangle with vertex at the origin and sides on the x-axis, the angle bisector, and the perpendicular line as shown in Figure 6. We hide the perpendicular line after constructing the triangle.
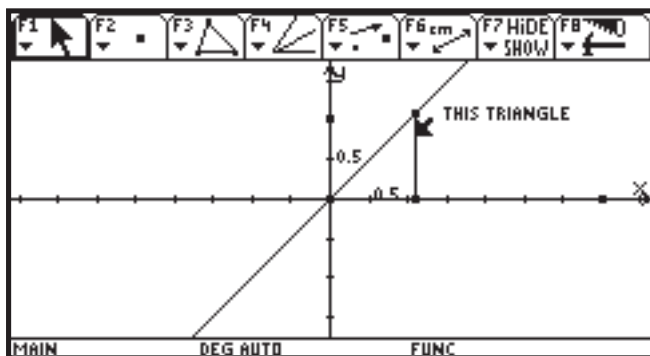

Figure 6. Constructing a triangle with CABRI geometry

Using the measurement feature we measure the distance of the leg of the triangle on the x-axis and the area of the triangle. We split the screen and open the data collection application as Figure 7 shows.
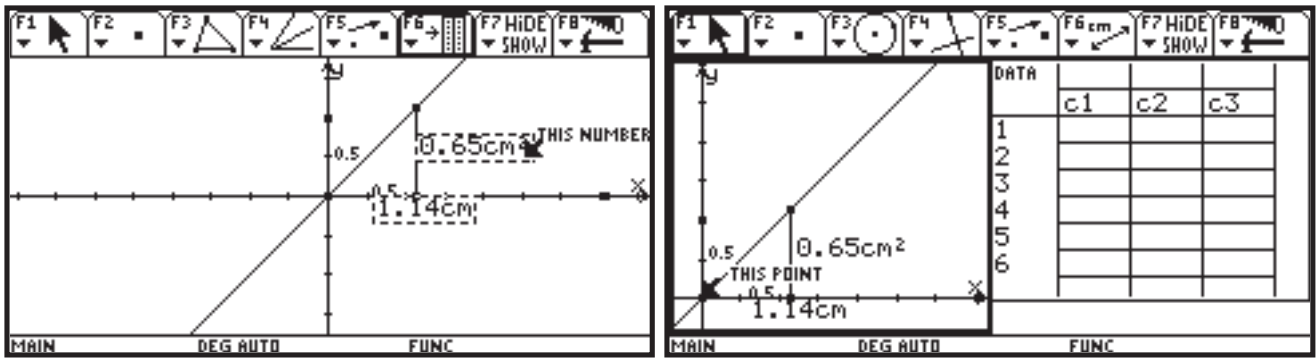
Figure 7. Measuring the length of one leg and the area of the triangle

The idea is to have the student think about the changing relationships between the length of the side along the x-axis and the area of the triangle. In the language of calculus, this is the area under the curve y = x from x=0 to x=a for some positive number a. Or, more formally, a function defined by the integral $A(x) = \int_0^x t\,dt$
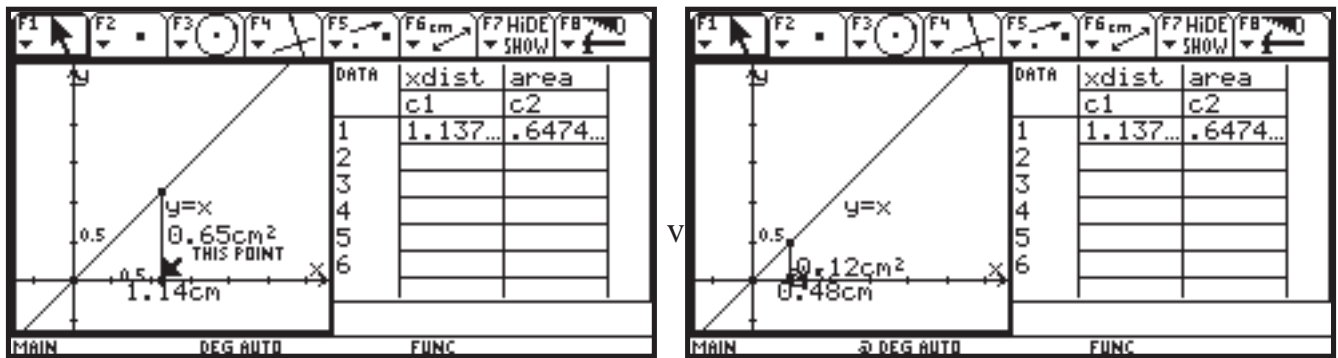

Figure 8. Sending the triangle measurement to the data table

Now we send the selected measurements to the data table by pressing ◆D and then drag the point shown and change the size of the right triangle (leg = 0.48 cm, area = 0.12 cm²). The selected measurements change automatically each time the dimensions of the triangle are changed. After each change we press ◆D to send the measurements to the data table as shown in Figure 9. These computer interactive geometry, measurement, and data "connections" are native to the TI-92.
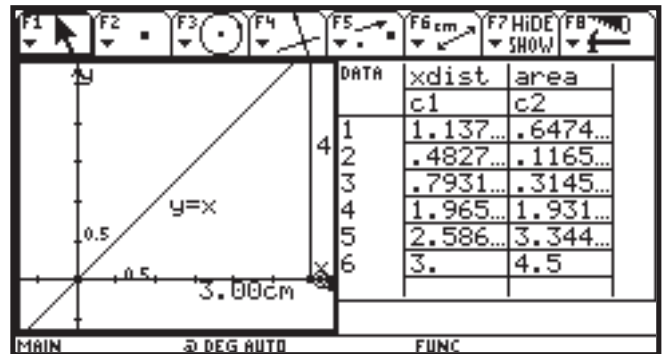

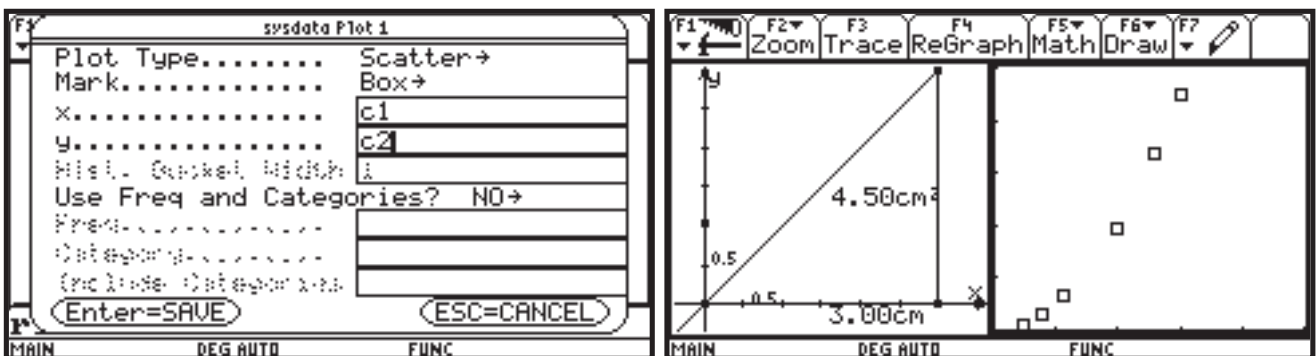**Figure 9. A data table of all the measurements**

Figure 10. Defining and plotting the triangle measurement data

Now we use the data analysis features of the TI-92 to define a scatter plot of the data (see Figure 10). Notice the plot is not linear. Here is an excellent opportunity for students to explore and find a graph of a function that contains the given data points. The graph of

y1= x^2 is an obvious first choice for most students. Of course, it will not contain the data points as shown in Figure 11. However, it is clear that a dilation of the graph of y1 = x^2 may fit the data.
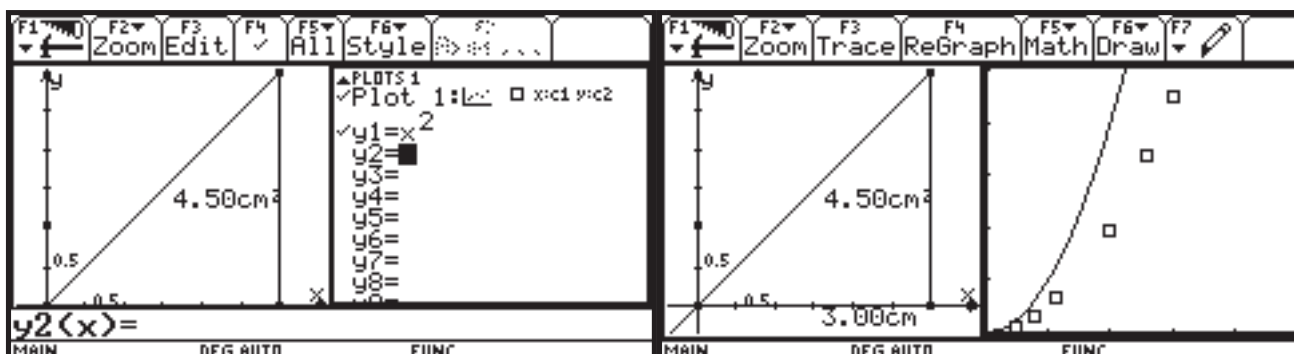

Figure 11. Finding the graph of a function that contains the data

A dilation by a factor of ½ will do the job. The graph of y1=$x^2/2$ is shown in Figure 12. You could also obtain the desired function by finding the quadratic regression model for the data (using a built-in algorithm on the TI-92).

Lower secondary students can use this example as a geometry and data analysis activity. Upper secondary students can go deeper into the functional relationships and calculus relationships by exploring the concept of a function defined by an integral and discovering that

$$A(x) = \int_0^x t\, dt = \frac{x^2}{2}$$

Extensions can be developed by finding the function determined by the area "under" the graph of y = k∗x and y = x^2 bounded by the lines x = 0, x = a, and the x-axis.

New sequence graphing functionality on the TI-92 facilitates more mathematics

The TI-92 sequence graphing software is more powerful and versatile than on previous graphing calculators. For example "deep" recursive sequence definitions are now easy and natural to construct.
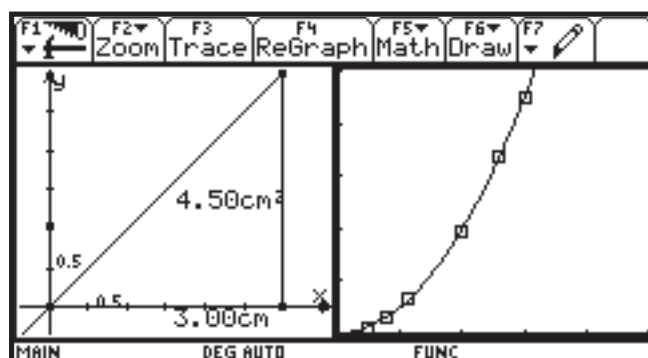

Figure 12. Graph of y1 = $x^2/2$ seems to contain the data

***Example 4:*** Graph and construct a table showing the values of the Fibonacci sequence.

Notice in Figure 13 that u1(n) is defined naturally as u1(n-1) + u1(n-2), the sum of the two previous terms. Also observe the initial conditions (two are required since the definition is "two deep") are given in the Y= editor as a simple list of two values (both have value 1 in this case). The split screen shows the table of values to the right. It can be scrolled to view any value desired. Also notice the graph with TRACE and that any term of the Fibonacci sequence can be evaluated directly on the home screen.
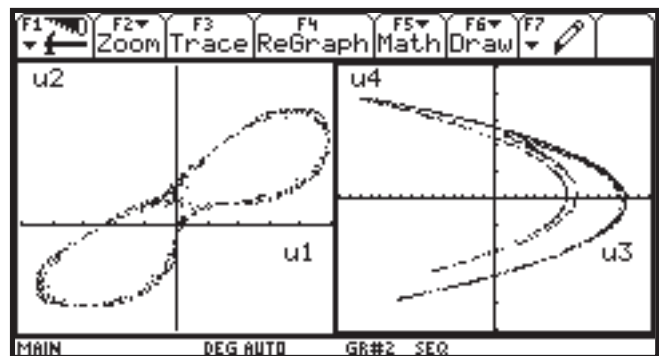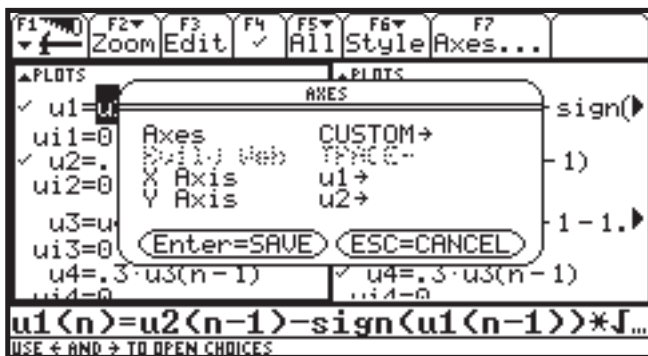
Figure 13

6
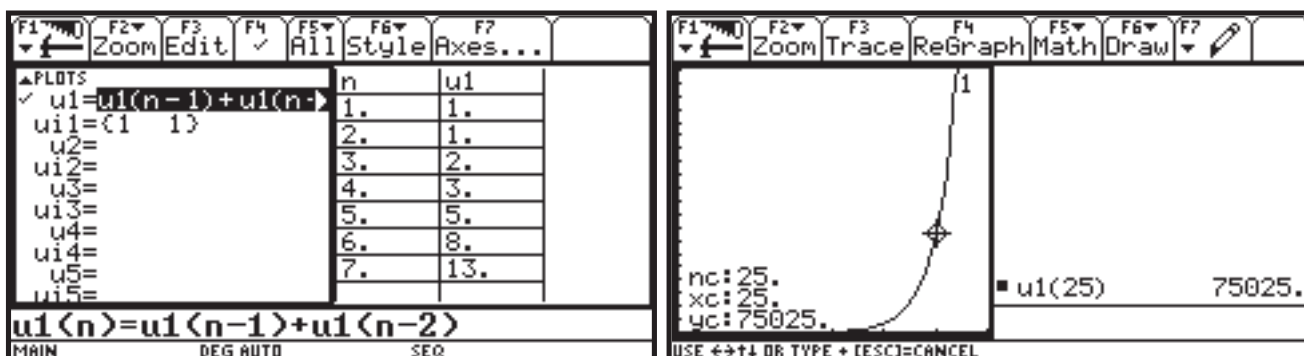
Another improvement is that the TI-92's sequence grapher allows the selection of a custom axes as shown in the next example from discrete dynamical systems theory.

Example 5: Graph the "strange attractors" given by Martin-Dewdmey and Henon. The Martin-Dewdmey map (plot) is given by u1 and u2 and the Henon map is given by u3 and u4 as shown in Figure 14.
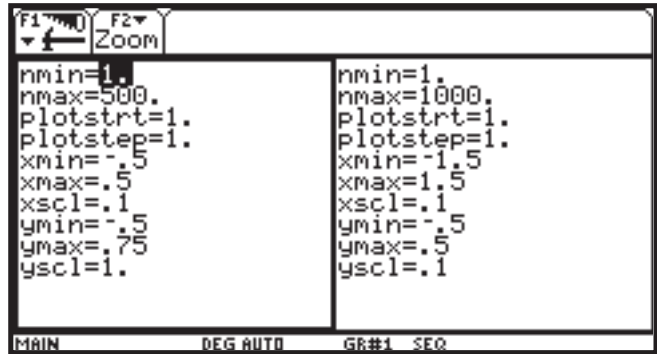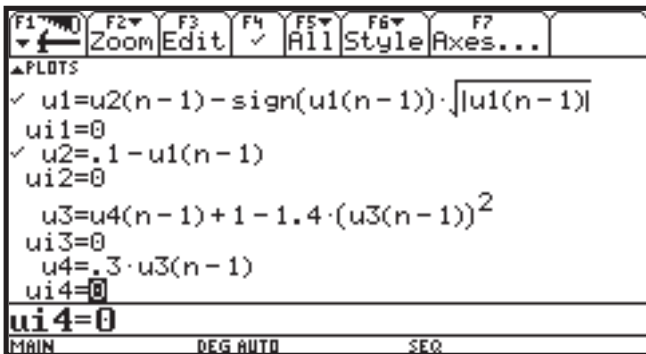
If these sequences were drawn as "time" plots (functions of t), you would see nothing but chaotic plots. No interesting pattern could be observed. However, the TI-92 allows the users to easily select the plotting axes so u2 and be plotted against u1 (and u4 against u3). The dramatic results (the strange attractors) are shown in Figure 15.

Creating new commands on the TI-92

There is no native eigenvalue command on the TI-92. Such a command is not needed because we can create one using other built-in computer algebra commands as follows. Figure 16 shows the result.

Example 6: Define a function (command) for finding eigenvalues of a matrix.

define eigenvl(a)=cZeros(det(a-x∗identity(dim(a)[1])),x)
or, more compactly,
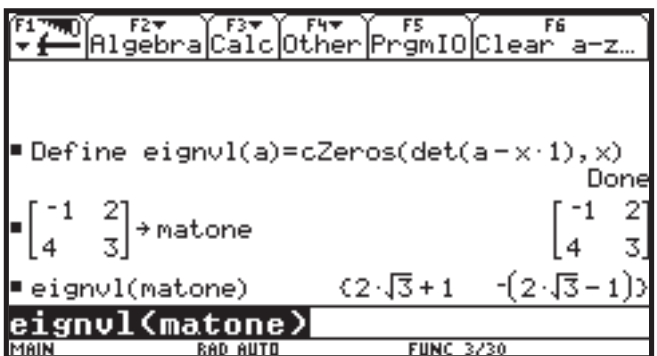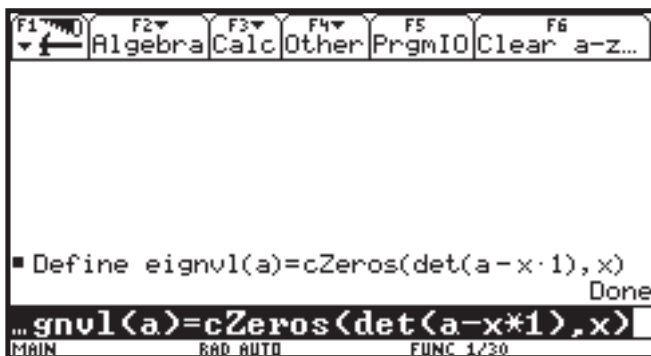$$= cZeros(det(a - x∗1), x)$$

Figure 16. Creating a user defined function: eigenvl

The left panel of Figure 17 shows the now "permanent" *eigenvl* command can be used with a square matrix of any dimension, and produces complex eigenvalues as well. The right panel shows the TI-92 computer algebra in action with simple native matrix algebra commands.
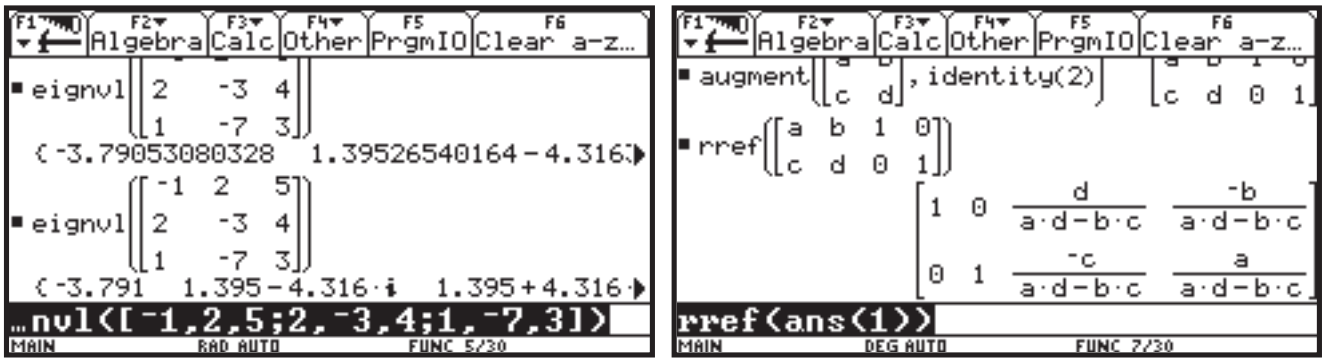
Figure 17. TI-92 matrix algebra commands in action

Visualizing the impossible!

The next example illustrates the power of the computer algebra system built-in on the TI-92 as well as the new 3D grapher. Here the technology helps the student to visualize *complex* zeros of polynomials in virtually the same way that we visualize real zeros of polynomials by associating them with x-intercepts of the graph of the polynomial.

Example 7: Graph the modulus surface associated with the polynomial $x^2 + 1$ and visualize the complex zeros of the polynomial.

The modulus surface associated with a polynomial f(x) is defined as the function of two variables given by $z(x, y) = abs(f(x + yi))$. The TI-92 deals with complex numbers naturally. The main idea is that just as all real zeros of a polynomial f(x) can be identified visually

by the intersection of the graph of y = f(x) and the x-axis, *all* complex zeros (including real ones of course) of the polynomial f(x) can be visually identified as the points where the modulus surface associated with the polynomial f(x) meets the xy plane. That is, where the graph of the surface $z(x, y) = abs(f(x + yi))$ touches the xy-plane.

In Figure 18, we first define f(x) to be the function x^2+1 (user defined functions are a new feature of the TI-92). Then the computer algebra software built-in on the TI-92 computes the expression $abs(f(x + yi))$. The result is the square root of a fourth degree polynomial in x and y!

The next series of figures shows the details of obtaining a 3D plot of the modulus surface associated with the simple polynomial x^2+1 with complex zeros i and -i.
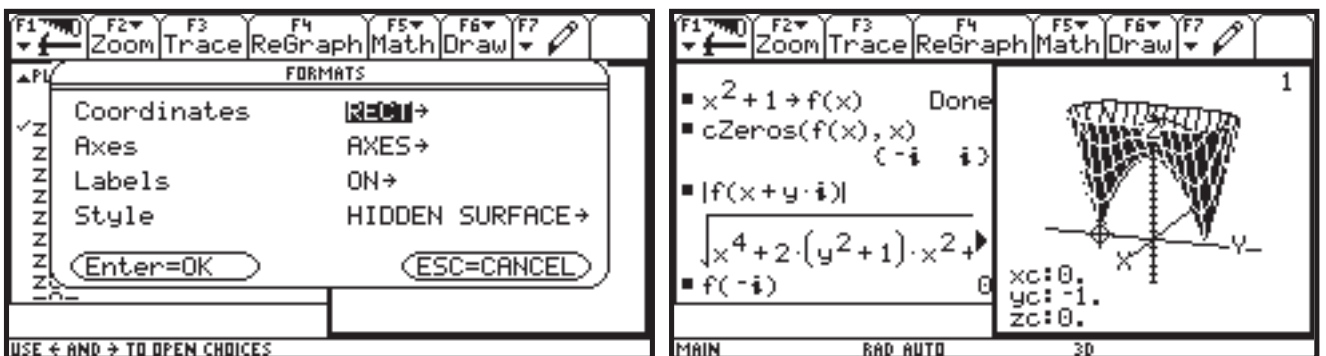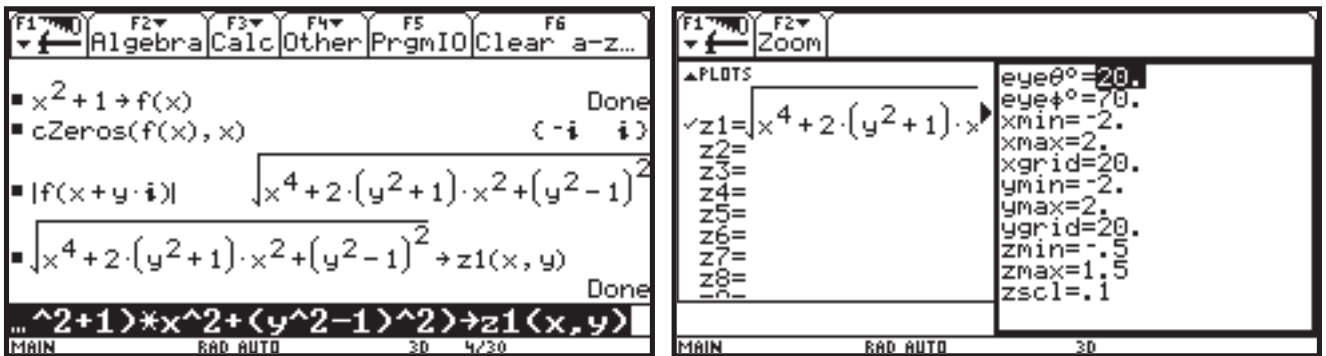

Figure 18. The "two tooth" modulus surface. Visualizing the complex zeros of $x^2 + 1$

8

The last panel of Figure 18 shows we can "see" the complex zeros i and $-i$ touching the xy plane at the points (0, -1) and (0, 1). The TRACE feature of the 3D grapher makes this connection even clearer!

The next series of figures helps us visualize the three complex zeros of the polynomial $x^3 - x^2 + x - 1$. The associated modulus surface function is the square root of a complicated sixth degree polynomial in x and y. Think about doing this algebraic computation with only paper and pencil. Try it!
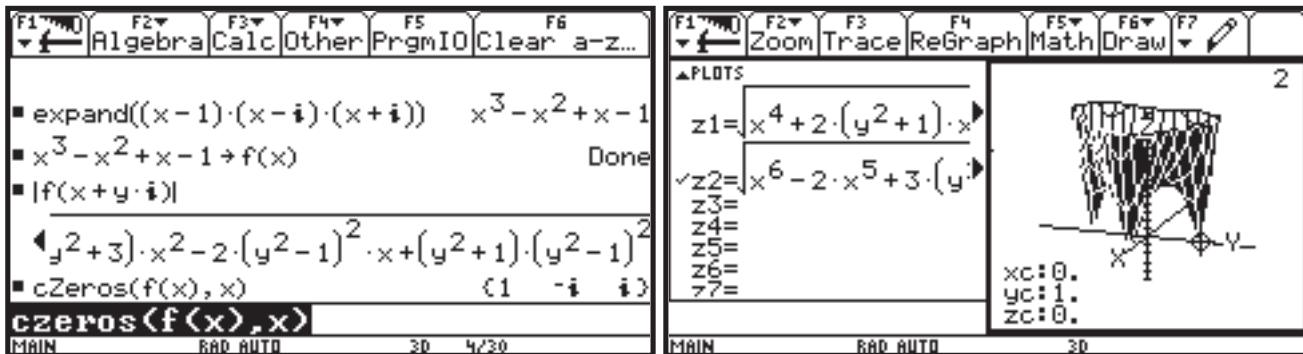

Figure 19. The "three tooth" modulus surface. Visualizing the complex zeros of $x^3 - x^2 + x - 1$

The three complex zeros of $x^3 - x^2 + x - 1$ (namely 1, -i, and i) are now easy to visualize as the points (1, 0), (0, -1), and (0, 1) on the associated modulus surface that touch the xy plane. Further details can be found in the delightful article by Long and Hern [Cliff Long and Thomas Hern, "Graphing the Complex Zeros of a Polynomial via its Modulus Surface", *The College Mathematics Journal*, 20 (1989) 98-105].

Concluding remarks

The TI-92's built-in integrated solvers, matrix algebra, data analysis, programming, and graphing software, along with its text editor, QWERTY keyboard, and large screen significantly extend the power of what is thought possible in a graphing calculator. More importantly, we believe mathematics instructors and students will find the DERIVE like computer symbolic algebra and CABRI II like computer interactive geometry systems built-in on the hand-held TI-92, to be powerful and very useful tools that will find extensive application in every secondary and beginning university mathematics course. These software packages were formally available only as expensive software for costly PC and Mac's.

Powerful new hand-held technology tools for enhancing mathematics teaching and learning, like the TI-92, will change the mathematics curriculum forever. It will take time as does all change. Mathematics instructors will be confronted with an increasing need to become technology literate as well as curriculum reformers. A major issue remaining in this decade will be to define and understand the diminishing role of teaching paper and pencil manipulative algebra and calculus. The need for pre-service and in-service teacher professional development blending mathematics curriculum reform with appropriate use of technology should be a top priority for educators in the next five years.