MAT221 Test 2: Vectors and Vector-valued functions

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (15 pts). Given the equation of the plane

$$2x + y - 3z = 6$$

1. (6 points) Carefully draw the plane in a suitable three dimensional coordinate system. Clearly indicate the coordinate axes x, y, and z, and the unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$.

2. (6 points) Find the point of intersection of this plane and the line l traced out by the vector-valued function $\mathbf{r}(t) = \langle 2 - 2t, 2 - t, 3t \rangle$.

3. (3 points) Carefully draw the line l in your figure, shown passing through the point you found in part 2 above and a second point on the line.

Problem 2 (18 pts). A few equations:

1. (6 points) Write the equation of the sphere of radius 2 with center (1,2,2).

2. (6 points) What is the equation of the intersection of the sphere above and the yz-plane?

3. (6 points) Write an equation for the plane passing through the points (0,0,1), (1,1,1), and (0,1,0).

Problem 3 (24 pts).

1. (10 points) Carefully draw the space curve with parametric equations given by

$$x(t) = t\cos(2t) \quad y(t) = t\sin(2t) \quad z(t) = t$$

for $-8 \le t \le 8$ from an eye position sighting along the vector $\mathbf{r} = \langle 1, 1, 1 \rangle$ towards the origin (that is, projecting onto the vectors $\mathbf{u}_1 = \langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \rangle$ and $\mathbf{u}_2 = \langle \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \rangle$).

2. (4 points) Show that the motion occurs on a cone.

3. (10 points) What is the equation of the line tangent to the curve when $t = \frac{\pi}{4}$?

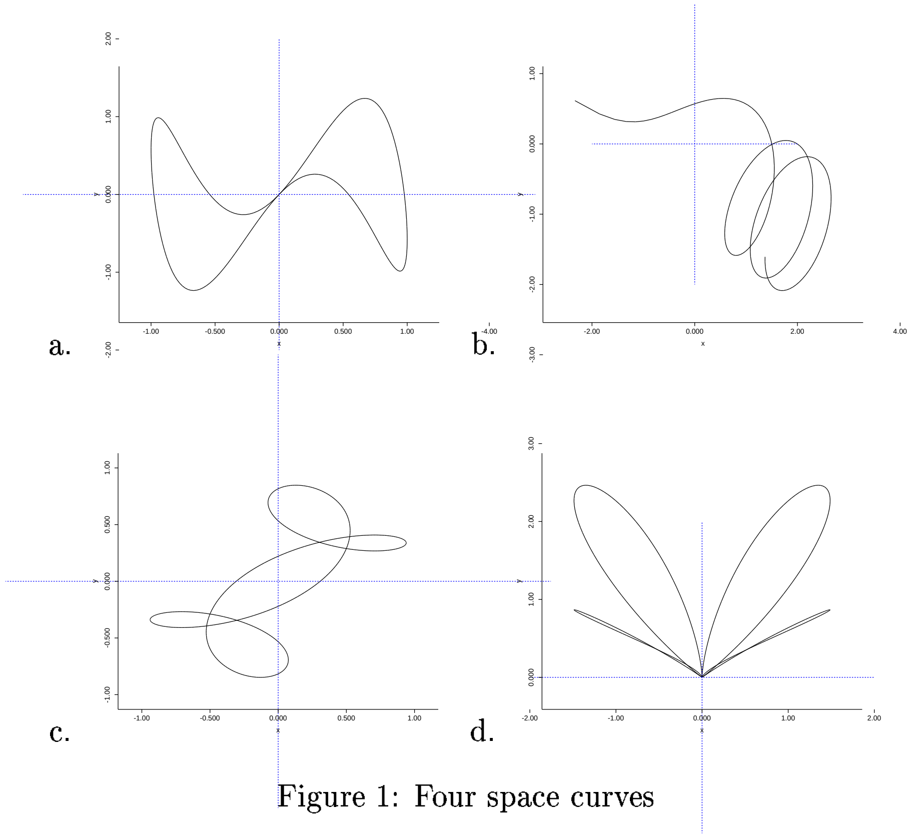

Figure 1: Four space curves

Problem 4 (16 pts). Match the space curves in Figure 1 to the parametric equations given below by placing the correct letter in the correct matching box for the equations. Provide some (mathematically sound!) rationale for your choices for full credit.

Notes: one of the sets of equations below has no corresponding plot! Leave its box empty.
The curves are projected with the eye position along the vector $\mathbf{r} = \langle 1, 1, 1 \rangle$.

1. ( )

$$x(t) = \sin(t) \quad y(t) = \cos(t) \quad z(t) = \sin(3t)$$

2. ( )

$$x(t) = \cos(t) \quad y(t) = \ln(t) \quad z(t) = \sin(t)$$

3. ( )

$$x(t) = \cos(2t) * \cos(t) \quad y(t) = \sin(2t) * \cos(t) \quad z(t) = \sin(t)$$

4. ( )

$$x(t) = (1 + \cos(4t))\cos(t) \quad y(t) = (1 + \cos(4t))\sin(t) \quad z(t) = (1 + \cos(4t))$$

5. ( )

$$x(t) = \tan(t)\cos(10t)\cos(t) \quad y(t) = \sqrt{1 - 0.25\cos(10t)}\sin(t) \quad z(t) = 0.5\cos(10t)$$

Problem 5 (10 pts). Given $\mathbf{r_1} = \langle -4, 3, 2 \rangle$ and $\mathbf{r_2} = \langle 0, 2, -1 \rangle$. Compute:

1. $\mathbf{r_1} \cdot \mathbf{r_1} =$

2. $\mathbf{r_1} \cdot \mathbf{r_2} =$

3. $|\mathbf{r_1}| =$

4. $\mathbf{r_1} \times \mathbf{r_1} =$

5. $\mathbf{r_1} \times \mathbf{r_2} =$

Problem 6 (17 pts). Consider

$$\mathbf{r}(t) = \langle \sin(t), \ln(t+1), e^t \rangle \times \langle t, t^2, 1 \rangle$$

1. (3 points) What is the domain of $\mathbf{r}$?

2. (4 points) What is the limit of $\mathbf{r}$ as $t \to 0$?

3. (10 points) Compute the derivative of $\mathbf{r}$ with respect to t.