MAT222 Final, Spring 2004:

Section 12.8-12, plus old stuff

Name:

Directions:

- All problems are equally weighted. The test is in two sections: you must skip two of the first seven problems (write "Skip" plainly on those you wish to skip); you must answer all five of the remaining problems (numbered eight to twelve).
- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).
- Indicate clearly your answer to each problem (e.g., put a box around it).
- **Good luck!**

Part I: old stuff – skip two of seven

Problem 1 Suppose you know that

$$\int_1^\infty \frac{1}{x^2}dx$$

is convergent. Based exclusively on that, what can you conclude about

1. $\int_1^\infty \frac{1}{x}dx$?

2. $\int_1^\infty \frac{1}{x^3}dx$?

What are the correct conclusions about the integrals above: are they convergent or not? Use any information at your disposal!

Problem 2 Use both the trapezoidal rule and Simpson's rule to approximate $\int_0^{\pi} \sin(x)dx$ (use $n = 8$). Compare both estimates to the true value – which is better?

Problem 3 Use integration by parts to integrate

1. $\int e^x x^2 dx$

2. $\int e^x \cos(x)dx$

Problem 4 Determine whether the following sequences converge or diverge; if one converges, find its limit (it is not sufficient to invoke a graph or calculator result to "determine" whether the sequences converge!).

1. $a_n = \ln(n+1) - \ln(n)$

2. $a_n = \frac{2n}{1-3n}$

3. $a_n = \frac{\ln(1+e^n)}{\sqrt{n}}$

Problem 5 Determine whether the following series are divergent, or absolutely or conditionally convergent (ditto what was said in Problem 4!):

1. $\sum_{n=1}^{\infty} \ln(\frac{n}{n+1})$

2. $\sum_{n=1}^{\infty} (-.1)^n$

3. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+4}$

Problem 6 Estimate the error in using the first five terms (through $n = 5$) to approximate

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

How many terms are necessary in order to ensure that the error be less than .001?

Problem 7 For which of the following is the ratio test inconclusive? Which, if any, are better evaluated using the root test?

1. $\sum_{n=1}^{\infty} \frac{1}{n^4}$

2. $\sum_{n=1}^{\infty} \frac{n}{3^n}$

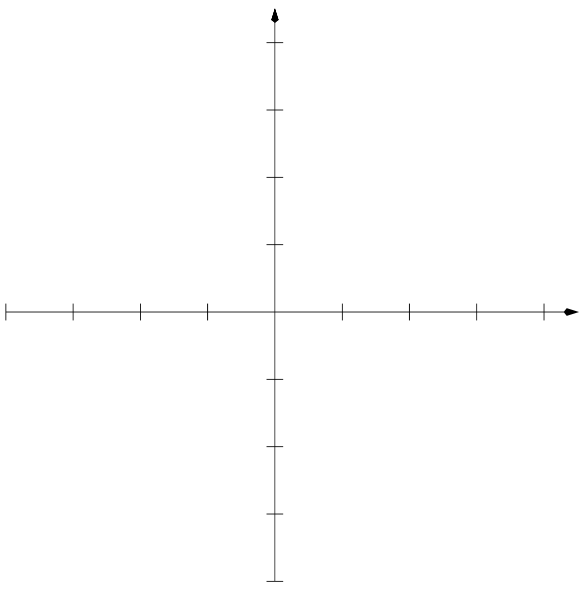
3. $\sum_{n=1}^{\infty} \frac{\ln(n)}{\sqrt{n}}$

4. $\sum_{n=1}^{\infty} \left(\frac{n+1}{2n^2+3}\right)^n$

Part II: new stuff (no skipping!)

Problem 8 Consider $f(x) = (x^2+1)e^x$: we seek to find an approximating polynomial P of 4^{th} degree that agrees well with f at the point $x = 1$. That is, the derivatives of the two functions P and f agree through fourth order. Find P.

Problem 9 Compute the 5^{th} degree Taylor polynomial T_5 for $f(x) = \tan^{-1}(x)$ at $x = 0$. Plot f and T_5 on the interval I=$[-2, 2]$ and describe the consequences of using T_5 to approximate f on the interval I.



Problem 10 Use the power series expansion for $1/(1-x)$ to get a power series for $\ln(1-x)$.

Problem 11 Express the function

$$f(x) = \frac{2+x}{1+x-2x^2}$$

as a power series by first using partial fractions. Determine the radius of convergence as well as the interval of convergence.

Problem 12 The fifth-degree Taylor polynomial $T_5(x)$ for $f(x) = e^x$ about zero is

$$e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} = \sum_{n=0}^{5} \frac{x^n}{n!}$$

1. Use the Taylor remainder theorem to bound the error incurred using this approximation on the interval $I = [-2, 2]$.

2. Graph the difference of f and T_5 on I, and use it to estimate the greatest error possible on I. How does it compare to your bound in part 1?