MAT220 Final: More on Series, and Recap of MAT220

Name:

Problem 1 (15 pts). Determine whether the following series are absolutely convergent, conditionally convergent, or divergent.

1.
$$\sum_{n=0}^{\infty} \frac{(-3)^n}{n!}$$

2.
$$\sum_{n=1}^{\infty} \frac{n^n}{3^{1+3n}}$$

3.
$$\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{5 \cdot 8 \cdot 11 \cdot \cdots \cdot (3n+2)}$$

Problem 2 (10 pts). Fix the following statements/definitions:

1. A series $\sum_{n=0}^{\infty} a_n$ is absolutely convergent if and only if $\lim_{n\to\infty} a_n = 0$.

2. In the root test, we check $\lim_{n\to 0}(|a_n|)^{\frac{1}{n}}$: if it's 0, then the series is absolutely convergent.

3. A series which is conditionally convergent has the property that rearranging terms in the summation automatically changes the sum of the series.

4. If a series is absolutely convergent, then it is certainly conditionally convergent.

Problem 3 (10 pts). For the following two power series, calculate the intervals and radii of convergence.

1.
$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{3^n}$$

2.
$$\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{(2n-1)!}$$

Problem 4 (15 pts). Given $f(x) = \ln(x)$. Use your TI as well as possible to do the following:

1. Approximate f by a Taylor polynomial with degree 3 centered at $a = 4$.

2. Use Taylor's inequality to estimate the accuracy of the approximation $f(x) \approx T_3(x)$ when x lies in the interval $3 \leq x \leq 5$.

3. Check your result in part (2) by graphing $|R_3(x)|$.

Problem 5 (10 pts). Using the power series for the function

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad (|x| < 1)$$

compute power series for the following functions. Show your work! Of course you could just use your TI, but don't! I want you to make use of the series above. Include the intervals of convergence.

1. $f(x) = \ln(1 + 2x)$
2. $f(x) = (1 + 2x)^{-3}$

■ Recap of Calc II ■

You should do eight (8) of the following ten (10) five-point problems. This means that you may skip two (2) of them. In large letters write "SKIP!" over the two you wish to skip.

Problem 6 (5 pts). Compute the integral $\int_1^e x\ln(x)dx$ by parts, showing all details.

Problem 8 (5 pts). Approximate (by hand - i.e., write out the coefficients, etc.) the integral

$$\int_{-1}^{1} \sqrt{1+x^3}\, dx$$

using Simpson's rule with $n = 8$.

Problem 14 (5 pts). Find a good bound on the error incurred by approximating the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(1+n)^2}$$

by the partial sum including all terms up to the term a_m.

Problem 15 (5 pts). Use the integral comparison test to demonstrate that the series

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

diverges.