

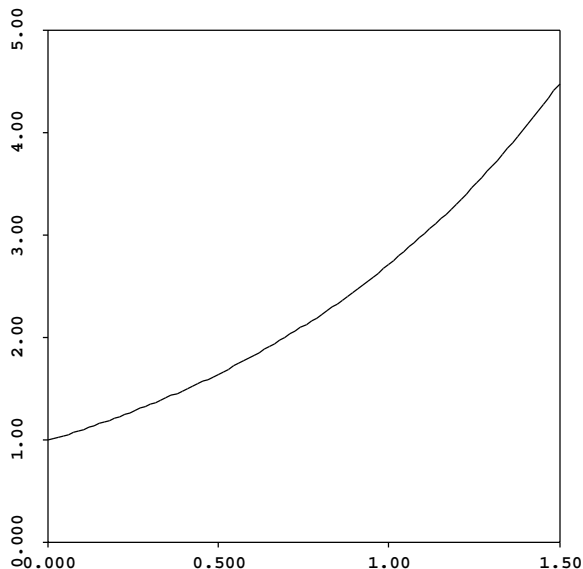
MAT222 Test 1: Integration, Sequences and Series

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (10 pts).

1. Draw the finite region bounded by the y-axis, and the graphs of the functions $f(x) = e^x$ and $g(x) = xe^x$.



2. To the right of the figure, write an integral which equals the area of the region from part 1.
3. In the space below calculate the integral given in part 2, using integration by parts. Show all steps!

Problem 2 (10 pts). Consider $I = \int_0^2 f(x)dx$, where f is the function whose graph is represented below. We want to use several approximations to estimate the area under the curve.

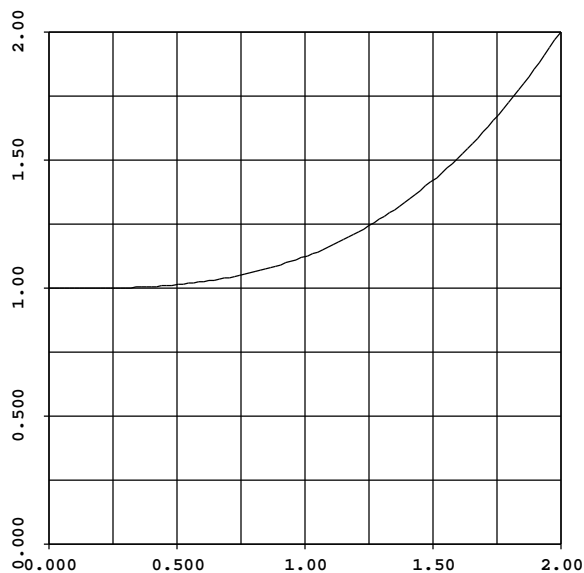


Table 1: x values, and corresponding function values $f(x)$

x	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
$f(x)$	1.00	1.00	1.02	1.05	1.12	1.24	1.42	1.67	2.00

1. In the space next to the graph, find the approximations L_4 , R_4 , T_4 , and M_4 .
2. For each of these methods, which are over- and which are under-estimates?
3. Calculate the value of S_8 .
4. If the function shown is that of $f(x) = 1 + x^3/8$, compare the approximations to the true value.

Problem 3 (10 pts). Integrate

$$\int \frac{x^2 + 2x}{x^2 + 2x - 3} dx$$

by partial fractions. Show all steps!

Problem 4 (10 pts). Consider the integral

$$\int_0^{\infty} \frac{e^{-x}}{x^{\frac{1}{2}}} dx$$

1. In what ways is this integral improper?
2. Demonstrate that this integral is convergent.

