MAT222 Test 2 (Spring 2004): Series

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (25 pts). Determine whether the following series are convergent or divergent (give valid reasons!). For those which are convergent, can you find the limit?

1.
$$\sum_{n=0}^{\infty} \frac{-1}{2(n+1)}$$

2.
$$\sum_{n=2}^{\infty} \frac{e^n}{n!}$$

3.
$$\sum_{n=1}^{\infty} \left(\frac{2n}{3n-1}\right)^n$$

4. The series with terms $a_0 = 5$ and

$$a_{n+1} = \frac{a_n}{2 - e^{-n}}$$

5.

$$\sum_{n=0}^{\infty} \frac{\sin(2n)}{(n+2)^2}$$

Problem 2 (10 pts). If we use the partial sum s_m to approximate

$$s = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

provide a (good!) upper bound for the error.

Problem 3 (10 pts). $\ln(2) = s = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

1. If we want to use the partial sum s_m to approximate s to an error of less than .001, how many terms m must one take?

2. Use your calculator to obtain this approximation, and check your answer!

Problem 4 (10 pts). Use a convergent series to demonstrate that

$$\int_1^{\infty} \frac{1}{2x^2 + \sin^2(x)} dx$$

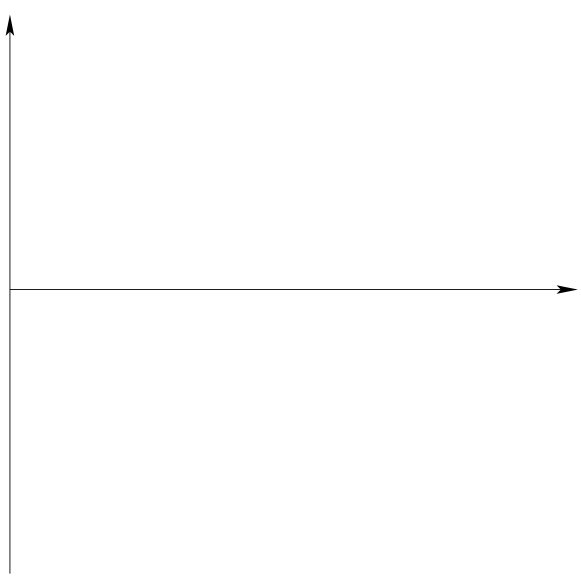
is convergent. Explain how the series justifies the conclusion that the integral is convergent.

Problem 5 (10 pts). Given $\{a_n\}$ where $a_0 = 1$ and $a_{n+1} = \frac{n}{n+1} a_n$. Test the convergence of $\sum_{n=0}^{\infty} a_n$.

Problem 6 (15 pts). Consider the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2^{n/2}} \tag{1}$$

1. Plot the first 10 terms of the series (1) and the first 10 partial sums of the series on the same axes below. Indicate clearly which graph is which.



2. Demonstrate that the series converges, and find the limit exactly (it is not sufficient to use your calculator only!).