MAT220 Test 3: Sequences and Series

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (30 pts). Determine whether the following sequences and series are convergent or divergent (give valid reasons!).

1.

$$\sum_{n=1}^{\infty} \frac{-3}{2^n}$$

2.

$$\left\{ \frac{n}{n+1} \right\}_{n=1}^{\infty}$$

3.

$$\sum_{n=2}^{\infty} \frac{n}{\ln(n)}$$

4.

$$\sum_{n=1}^{\infty} \cos(n)$$

5.

$$a_n = \ln(n+1) - \ln(n)$$

6.

$$\sum_{n=2}^{\infty} \frac{2n^2 + 7n}{3^n(n^2 + 5n - 1)}$$

Problem 2 (10 pts). Analyze the integrals

$$\int_0^1 \frac{dx}{x^p} \quad \text{and} \quad \int_1^\infty \frac{dx}{x^p},$$

determining conditions on p for which they converge.

You should use the *definition* of each improper integral to do your analysis (that is, use limits).

Problem 3 (10 pts). Give examples of series for which

1. $a_n \to 0$ as $n \to \infty$, but $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges
2. $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, but $\displaystyle\sum_{n=1}^{\infty} \sqrt{a_n}$ diverges
3. $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, but $\displaystyle\sum_{n=1}^{\infty} |a_n|$ diverges
4. $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, but $\displaystyle\sum_{n=1}^{\infty} a_n^2$ diverges

Problem 4 (10 pts).

1. State in precise terms (i.e., using ε!) what it means to say that the sequence $\{a_n\}$ converges to the limit L.

2. From your definition (and *not* using the algebra of limits) prove that

$$\frac{n}{2n+7} \to \frac{1}{2} \text{ as } n \to \infty.$$

Problem 5 (10 pts). Decide whether the following integrals are convergent or divergent (give reasons!):

1.

$$\int_0^4 \frac{dx}{x^2 + x - 6}$$

2.

$$\int_1^\infty \frac{dx}{x^2 + e^x}$$

Problem 6 (5 pts). Find the values of p for which the series is convergent:

$$\sum_{n=1}^{\infty} n(1 + n^2)^p$$

Problem 7 (15 pts). Consider the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\frac{1}{2}}} \tag{1}$$

1. Demonstrate that the series converges, and use your calculator to estimate the limit.

2. Plot the first 10 terms of both the sequence $\left\{\frac{(-1)^{n-1}}{n^{\frac{1}{2}}}\right\}_{n=1}^{\infty}$ and the series (1) on the same axes. Indicate clearly which graph is which.

3. How many terms of the series (1) do we need to add in order to find the sum to an accuracy of $< .0001$?

Problem 8 (10 pts). Consider the series

$$\sum_{n=1}^{\infty} \frac{1}{n^3} \tag{2}$$

1. Use the sum of the first 20 terms to estimate the sum of the series.

2. Bound the error of this estimate (that is, find values a and b such that $a \leq R_{10} \leq b$ "tightly").