MAT227 Exam 1 (Spring 2015)

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1: (25 pts) Consider the integral

$$I = \int_0^6 (x^2 + 1)dx$$

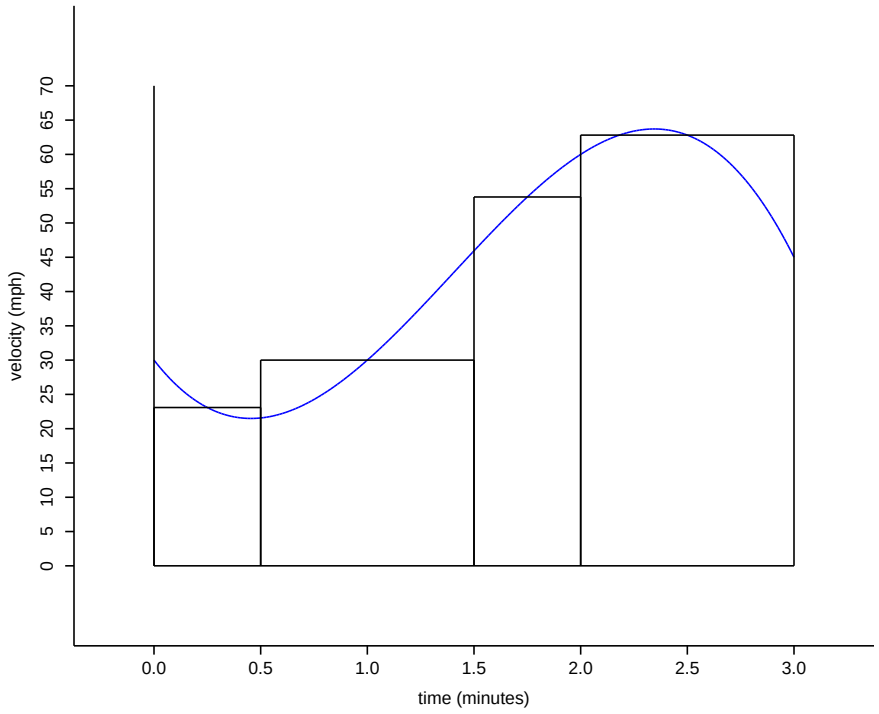
a. (5 pts) Show that the exact value of this integral is 78, using the fundamental theorem of calculus.

b. (15 pts) Using 3 equally spaced sub-intervals, approximate the integral. Compute the Simpson's rule approximation, by first computing the right, left, trapezoidal, and midpoint approximations. Show your work!

Method	Estimate	Error (estimate - true)
True	78	NA
LRR		
RRR		
Trapezoidal		
Midpoint		
Simpson		

c. (5 pts) Examine the errors. Are the results consistent with what we should expect? (**Be specific.**)

Problem 2: (20 pts) The continuous graph in this figure represents the velocity of a car over a 3-minute span, in mph.



a. (10 pts) Use the Riemann sum represented in the figure to estimate the distance traveled. Pay attention to units.

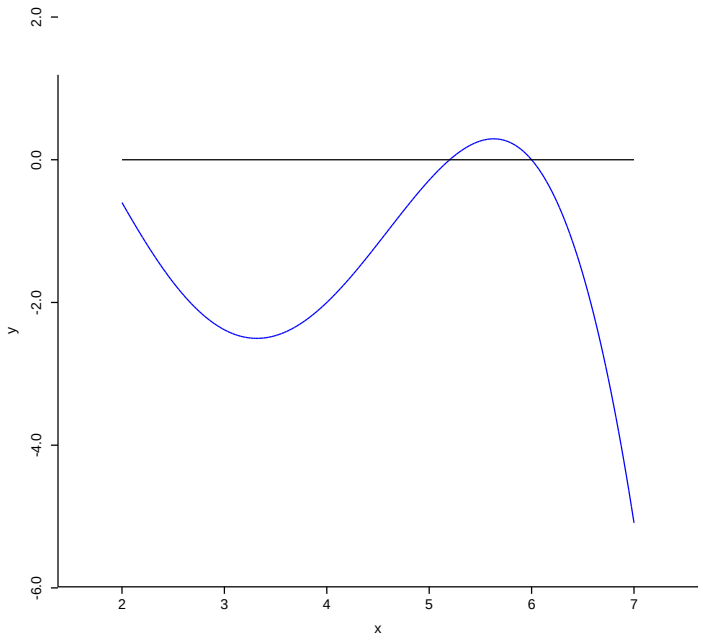
b. (6 pts) Identify the partition and the set of intermediate points ("centers") used in the Riemann sum.

   i. P=

   ii. C=

c. (4 pts) Show the area estimated using the **trapezoidal** method by **carefully** drawing an appropriate graph on the figure. (You do not need to compute the estimate.)

Problem 3: (20 pts) Let $A(x) = \int_2^x f(t)dt$, with the graph of f represented in the following figure:



a. (4 pts) True or false (with reasons!): $A(x) \leq 0$ for all x on the interval shown.

b. (6 pts) Estimate (and graph) the values of $A(x)$ at integer values of x from 2 to 7.

c. (4 pts) Where does $A(x)$ have a local minimum? Where does $A(x)$ have a local maximum?

d. (6 pts) Consider $h(x) = \int_2^{x^2} f(t)dt$. Write a formula for $h'(x)$ in terms of A, f, and x.

Problem 4: (20 pts) Consider $\int_0^2 (7+4x)dx$.

a. (15 pts) Use the definition of an integral as a Riemann sum to evaluate the integral.
(Hint: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.)

b. (5 pts) Use the fundamental theorem to evaluate the integral, and compare your answers.