

## MAT227 Test 2 (Spring 2015): Integrals, Exponentials, and Inverses

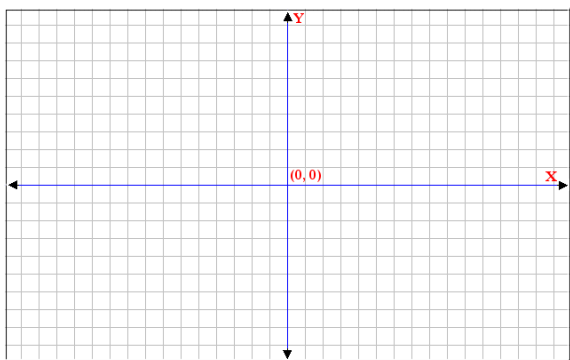
Name:

**Directions:** Problems are not equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

**Problem 1:** (10 pts) Evaluate  $I = \int_1^2 \frac{e^{x-2}}{x^3} dx$ . Show and justify every possible step. A correct answer alone is worth 1 point.

**Problem 2:** (20 pts) Consider the region bounded by the function  $f(x) = 8\left(\frac{x}{15}\right)^2$  and the function  $g(x) = 2 + \frac{2}{5}x$  (for  $x > 0$ ), and the  $y$ -axis.

- a. (4 pts) Draw the region, indicating which function is which, and shade in the physical area between the two curves. Label your graph!



- b. (4 pts) Estimate the area between the two curves. Explain your estimate.

- c. (6 pts) Write down and evaluate an integral that represents the area between the two curves.

- d. (6 pts) Write down (but do not evaluate) an integral that represents the volume generated by rotating this region about the  $x$ -axis.

**Problem 3:** (20 pts) Variety pack – show work!

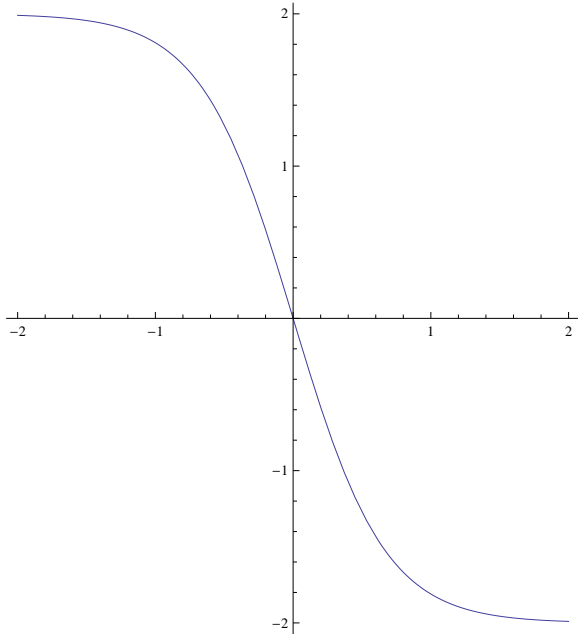
a. Solve for  $x$ :  $\ln(x^2 + 1) - 2\ln(x) = 1$

b. Simplify:  $\log_{10}(\sqrt{40})$

c. There is no “ $\log_3$ ” button on your calculator. Describe how one might rewrite function  $f(x) = \log_3(x)$  so that one may compute values of  $f$  using a calculator.

d. Compare the growth rates of the functions  $\ln(x)$  and  $x^{0.1}$  as  $x$  gets large. You may use your recollections of our work in class.

**Problem 4:** (20 pts) Consider the function  $f(x) = \frac{4}{1 + e^x} - 2$ , whose graph is shown below:



- (8 pts) Carefully (and **I mean carefully!**) graph the inverse function  $f^{-1}(x)$ . Sloppy and rough (but reasonably correct) gets you only 2 points.
  - (8 pts) Find an explicit formula for the function  $f^{-1}(x)$ .
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- (4 pts) What are the domains and ranges of the function  $f$  and its inverse?