Polar Coordinates

MAT 229, Spring 2025

Supporting materials

If you wish to get a different perspective on the notes below, try either of the following textbook sections.

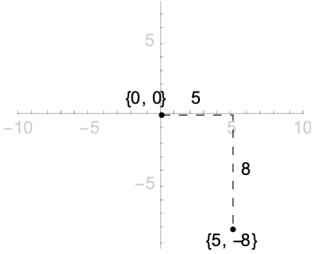
- Strang's *Calculus*
  Section 7.3: Polar coordinates
- Stewart's *Calculus*
  Section 10.3: Polar coordinates
- Boelkins/Austin/Schlicker's *Active Multivariable Calculus*
  Section 11.5.1: Polar coordinates

Different coordinate systems

The Cartesian coordinates for a point in the plane is like an address for where the point is located: we use the coordinates to determine how to get to the given point.

Example

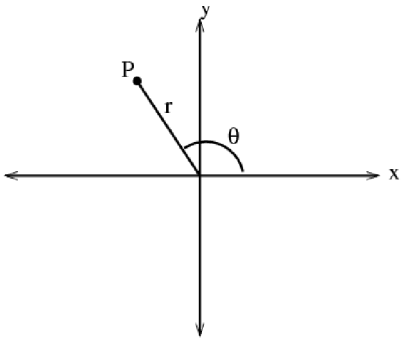
Oklahoma. Lots of cities' streets are laid out along grids (especially in the flatlands, as seen in this street map of Tulsa). We talk of "blocks", and if you are at the origin (e.g. the corner of Main and Wooster), you get to the point $(5, -8)$ by travelling 5 blocks in the positive x direction (usually the east), then go 8 blocks in the negative y direction (usually south):



Polar Coordinates

Polar coordinates is a different addressing scheme for points in the plane that give you information on how to travel from the origin to the given point. Polar coordinates take the form (r, θ) where

θ represents the angle from the positive x-axis to head along;
r is the distance to travel along that heading.



For example, $(r, \theta) = (4, \pi/6)$ is the point one can reach by starting at the origin and walking 4 units along the ray that is 30° north of the positive x-axis.

Questions

- Plot the point A given by polar coordinates $(r, \theta) = (2, \pi)$.
- Plot the point B given by polar coordinates $(r, \theta) = (3, -\pi/4)$.
- How does the point C with $(r, \theta) = (1, \pi/3)$ compare with D, $(r, \theta) = (1, \pi/3 + 2\pi)$?
- In general, how does (r, θ) and $(r, \theta + 2\pi)$ compare?
- What might a negative value for r mean? Plot E with $(r, \theta) = (-4, \pi/6)$.
- How does the point F with $(r, \theta) = (2, \pi/6)$ compare with G of $(r, \theta) = (-2, \pi/6 + \pi)$?
- In general, how does $(-r, \theta)$ and $(r, \theta + \pi)$ compare?

Polar and Cartesian coordinates

Every point in the plane has both Cartesian coordinates and polar coordinates. How are they related?

- Write x and y in terms of r and θ. Write r and θ in terms of x and y.
- Find Cartesian coordinates for the point with polar coordinates $(1, \pi/3)$.
- Find polar coordinates for the point with Cartesian coordinates $(2, 1)$. Find polar coordinates for the point with Cartesian coordinates $(-2, -1)$.

Polar curves

Typically, polar curves are represented by equations of the form

$$r = f(\theta)$$

where θ varies over a range of values.

Question

What is the curve given by $r = 5, 0 \le \theta \le 2\pi$?

Question

Plot the polar curve given by $r = 2 - \cos(\theta)$ for $0 \le \theta \le 2\pi$ by making a table of values and plotting points

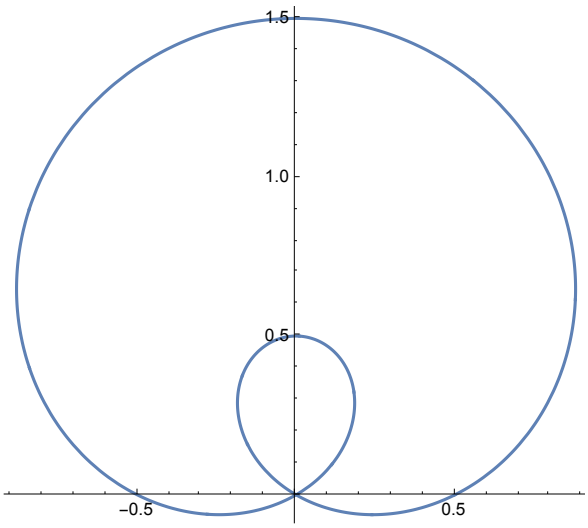
θ	$r = 2 - \cos(\theta)$
0	□
$\pi/8$	□
$\pi/4$	□
$3\pi/8$	□
$\pi/2$	□
$5\pi/8$	□
$3\pi/4$	□
$7\pi/8$	□
π	□
$9\pi/8$	□
$5\pi/4$	□
$11\pi/8$	□
$3\pi/2$	□
$13\pi/8$	□
$7\pi/4$	□
$15\pi/8$	□
2π	□

Questions

- Identify the curve $r = 4\sin(\theta)$ by converting it to a Cartesian equation.
- Identify the curve $r = \frac{5}{\cos(\theta)}$ by converting it to a Cartesian equation.

Questions

The polar curve $r = \frac{1}{2} + \sin(\theta)$ is shown below.



- The origin is a point on this curve. What would be the r-value of this point?
- There are two angles associated with this point. What are they?

Question

The two polar curves $r = 1/2$ and $r = -\sin(\theta)$ are shown below. What are polar coordinates for their points of intersection?