Exam 1: MAT 229, Spring 2025 Name:

Show your work to receive credit; cross out (don't erase).

Draw your graphs on this sheet. Please carefully separate problems on your answer sheets.

1. (18 pts) Compute the limits:

1.1. $\lim_{x\to 0} \frac{\sin(2x)}{\tan(3x)}$

1.2. $\lim_{x\to 0} x \ln(x+1)$

1.3. $\lim_{x\to \infty} x\, e^{-3x}$

2. (20 pts) Compute the derivatives:

2.1. $f(x) = \arctan(\sin(x))$

2.2. $f(x) = \ln\left(2\, e^{x+1}\right)$

3. (30 pts) Compute the integral using an appropriate technique of integration:

3.1. $\int x \cos(2x)\, dx$

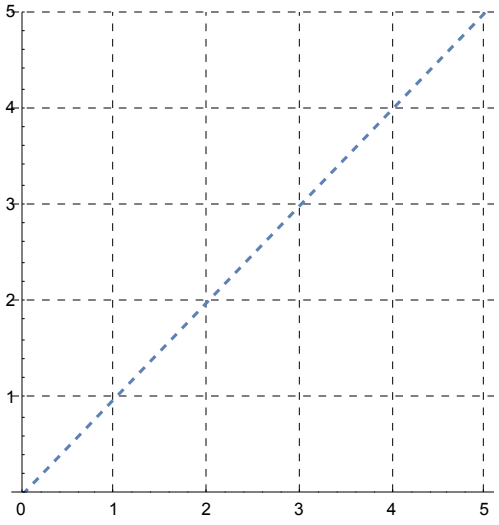
3.2. $\int \tan(x) \sec^4(x)\, dx$

3.3. $\int_{-1}^{1} \frac{x}{\sqrt{4-x^2}}\, dx$

4. (12 pts) Function f is invertible. Given that $f(3)=4$, and that $f'(3)=2$,

4.1. write the equations of the tangents line to f at $x=3$ and to f^{-1} at $x=4$, in point-slope form (8 pts).

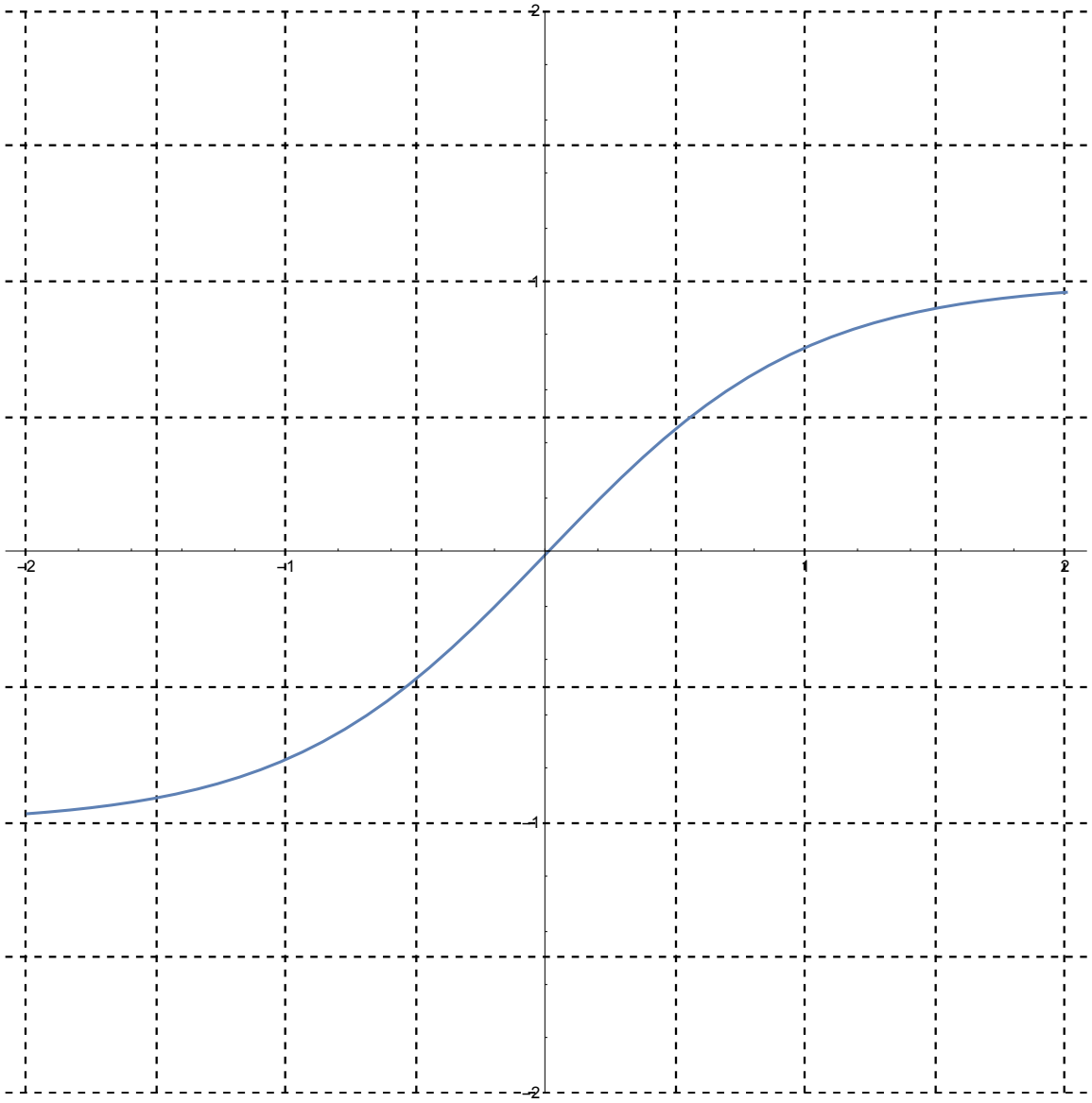
4.2. Draw both tangent lines here (4 pts):



(20 pts) The hyperbolic tangent is defined as

$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1} = y.$

Its graph is shown here:



5.1. Tanh is invertible: graph its inverse on the plot above (6 pts).

5.2. Find a formula for its inverse, and give its domain and range (8 pts).

5.3. Compute the derivative of the inverse (6 pts). (The derivative of tanh(x) is $\text{sech}^2(x)$, and $1 = \cosh^2(x) - \sinh^2(x)$.)

6. Extra Credit (5 pts): Use the limit definition of the derivative ("the most beautiful idea in calculus") to compute the derivative of $f(x) = e^x$. You may use the fact that the base e was chosen because its graph has a slope of 1 at x=0.