Exam 1 -- Lab portion
(1/3 of total exam grade)

MAT 229, Spring 2025

Name: Wyatt Ethier

Great work, Wyatt: I've got nothing else to say! :)
Prof. Long

Instructions:

In this portion of the exam, you will study a function, called "f", defined in the cell below:

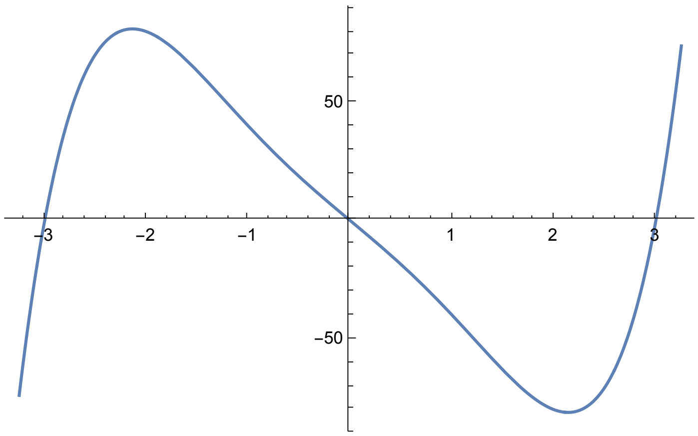
```
In[1]:= f[x_] := -36 x - 5 x^3 + x^5
```

- "Show your work": document your thoughts or any issues encountered in the notebook (using comments or text cells).
- If you get stuck, don't panic. Ask me a question.
- At the end of the hour, please email your final notebook to me at longa@nku.edu. I should be able to execute your file and see the solutions clearly.
- Featured commands:
  - Plot (Options: PlotRange, PlotStyle)
  - Solve (Options: Real)
  - ListPlot (Options: PlotStyle)
  - Show
  - Integrate

Exercises:

1. Plot f on the interval [-3.25,3.25] (5 pts).

In[2]:=
```
Plot[f[x], {x, -3.25, 3.25}]
```

Out[2]=


2. Find the local extrema of f by solving for where the derivative of f is equal to 0. Numerical approximations are fine, and you should **ignore any imaginary or complex roots** (8 points). Create a list "extrema" of the pairs {x,f(x)} (2 points)

In[3]:=
```
fp[x_] = D[f[x], x]
solutions1 = Solve[fp[x] == 0, x]
```

Out[3]= $-36 - 15 x^2 + 5 x^4$

Out[4]= $\left\{\left\{x \to -i \sqrt{\frac{3}{10}\left(-5+\sqrt{105}\right)}\right\}, \left\{x \to i \sqrt{\frac{3}{10}\left(-5+\sqrt{105}\right)}\right\}, \left\{x \to -\sqrt{\frac{3}{10}\left(5+\sqrt{105}\right)}\right\}, \left\{x \to \sqrt{\frac{3}{10}\left(5+\sqrt{105}\right)}\right\}\right\}$

In[5]:=
```
min = x /. solutions1[[4]]
max = x /. solutions1[[3]]
extrema = {{max, f[max]}, {min, f[min]}}
```

Out[5]= $\sqrt{\frac{3}{10}\left(5+\sqrt{105}\right)}$

Out[6]= $-\sqrt{\frac{3}{10}\left(5+\sqrt{105}\right)}$

Out[7]= $\left\{\left\{-\sqrt{\frac{3}{10}\left(5+\sqrt{105}\right)}, \frac{3}{2}\sqrt{\frac{3}{10}}\left(5+\sqrt{105}\right)^{3/2} - \frac{9}{100}\sqrt{\frac{3}{10}}\left(5+\sqrt{105}\right)^{5/2} + 18\sqrt{\frac{6}{5}\left(5+\sqrt{105}\right)}\right\}, \left\{\sqrt{\frac{3}{10}\left(5+\sqrt{105}\right)}, -\frac{3}{2}\sqrt{\frac{3}{10}}\left(5+\sqrt{105}\right)^{3/2} + \frac{9}{100}\sqrt{\frac{3}{10}}\left(5+\sqrt{105}\right)^{5/2} - 18\sqrt{\frac{6}{5}\left(5+\sqrt{105}\right)}\right\}\right\}$

In[8]:=

3. Find the inflection points of f by solving for where the second derivative is equal to 0. Numerical approximations are fine, and you should ignore any imaginary or complex roots (8 points). Create a list “inflection” of the pairs {x,f(x)} (2 points)

In[9]:=
```
fpp[x_] = D[fp[x], x]
solutions2 = x /. Solve[fpp[x] == 0, x]
```

Out[9]= $-30x + 20x^3$

Out[10]= $\left\{0, -\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}\right\}$

In[11]:=
```
inflection = {{solutions2[[1]], f[solutions2[[1]]]},
  {solutions2[[2]], f[solutions2[[2]]]}, {solutions2[[3]], f[solutions2[[3]]]}}
```

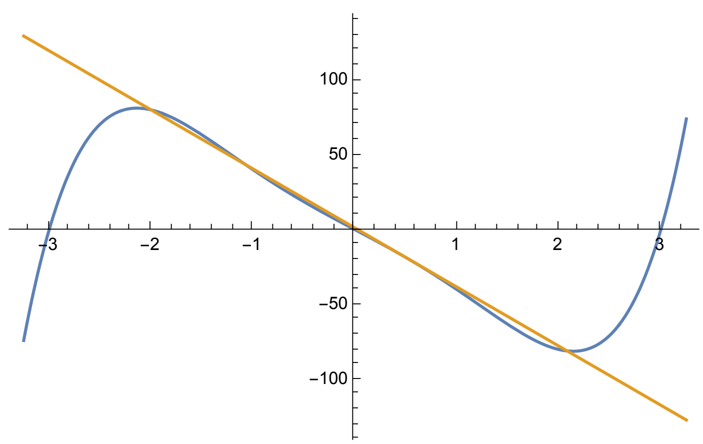
Out[11]= $\left\{\{0, 0\}, \left\{-\sqrt{\frac{3}{2}}, \frac{21\sqrt{\frac{3}{2}}}{4} + 18\sqrt{6}\right\}, \left\{\sqrt{\frac{3}{2}}, -\frac{21\sqrt{\frac{3}{2}}}{4} - 18\sqrt{6}\right\}\right\}$

4. Define the local linearization to f at $x = \frac{1}{2}$ (whose graph is the tangent line when $x = \frac{1}{2}$). Call it locallinear. (10 points)

In[12]:=
```
tangentF[x_] = fp[1 / 2] (x - 1 / 2) + f[1 / 2] (*defining the tangent line*)
Plot[{f[x], tangentF[x]}, {x, -3.25, 3.25}] (*plotting to check my work*)
```

Out[12]= $-\frac{595}{32} - \frac{631}{16}\left(-\frac{1}{2} + x\right)$

Out[13]=



In[14]:=
```
locallinear = tangentF[x]
```

Out[14]= $-\frac{595}{32} - \frac{631}{16}\left(-\frac{1}{2} + x\right)$

5. Define a function called g, which is the antiderivative of f whose value is 0 at $x = 0$. (10 points)

In[15]:=
```
g[x_] = Integrate[f[x], x]
g[0]
```

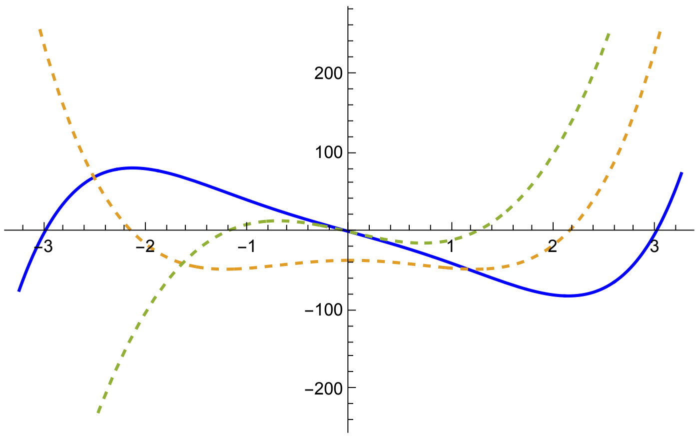
Out[15]= $-18\,x^2 - \frac{5\,x^4}{4} + \frac{x^6}{6}$

Out[16]= 0

6. Provide four graphs:

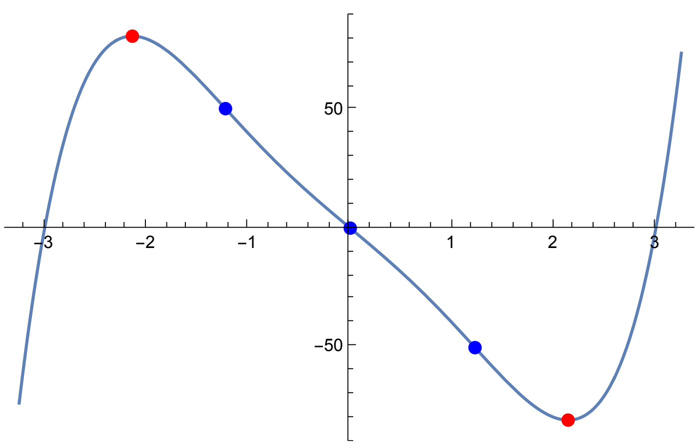
a. A graph of f and its first and second derivatives on the interval [-3.25, 3.25] (8 points). Make the graph of f Blue and the two derivatives Dashed (2 points).

In[17]:=
```
Plot[{f[x], fp[x], fpp[x]}, {x, -3.25, 3.25}, PlotStyle → {Blue, Dashed, Dashed}]
```

Out[17]=


b. A graph of f with points indicating the locations of the extrema and the inflection points on the interval [-3.25, 3.25] (8 points). Make the extreme points Red and the inflection points Blue (2 points). **Extra Credit**: make the points Large for 3 points.

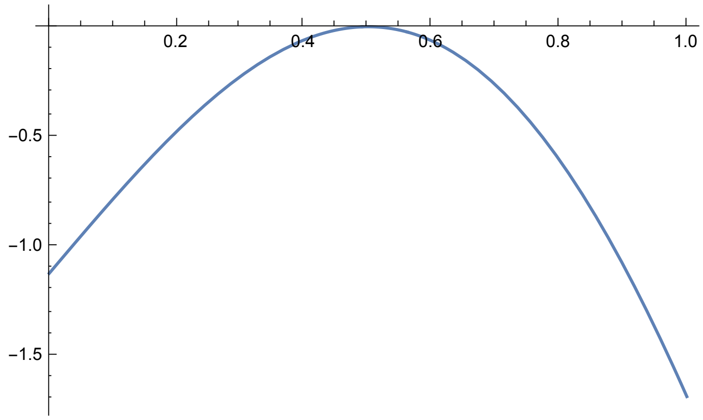
In[18]:=
```
Show[
 Plot[f[x], {x, -3.25, 3.25}],
 ListPlot[extrema, PlotStyle → {Red, PointSize[Large]}],
 ListPlot[inflection, PlotStyle → {Blue, PointSize[Large]}]
]
```

Out[18]=


c. A graph of the difference between f and locallinear on the interval [0,1] (8 points). Comment on how well the local linearization approximates f on this interval (2 points).

In[19]:=
```
Plot[f[x] - locallinear, {x, 0, 1}]
(*not very well because it has about 1.5 range of error*)
```

Out[19]=



d. A graph of f and its antiderivative g on the interval [-3.25, 3.25] (8 points). Make the range [-200, 100] (2 points).

In[20]:=
```
Plot[{f[x], g[x]}, {x, -3.25, 3.25}, PlotRange → {-200, 100}]
```

Out[20]=