Review for Exam 1

MAT 229, Spring 2025

Format

- Questions will be similar to homework questions and quiz questions.
- On the pen and paper part you must **show your work**. For example, if you need to evaluate a definite integral, you must derive the antiderivative and express how you evaluate it.
- When you believe that you've done something incorrect, **cross out rather than erase.** That way I can give you partial credit if you were actually doing useful work (but perhaps got off the path).
- You may have a one-page (front and back) sheet for formulas, etc. on Thursday (recall that we have a techno-free test on Thursday, and then Mathematica test on Friday). You may use copies of your labs for Friday.

Topics:

- From Calculus 1:
  - Domains and ranges of functions
  - Transformations of a function
  - Tangent lines
  - Absolute max/min
  - Intervals of increase/decrease and local max/min
  - Intervals of concavity and inflection points
  - Limits, and **the limit definition of the derivative**
  - derivative rules, such as product rule, chain rule
  - Fundamental Theorem of Calculus
  - Substitution for integrals
  - Calculations of Area
  - Volumes of solids of revolution
- Classes of functions
  - Polynomials
  - Rational functions (ratios of polynomials)
  - Root functions and power functions (e.g. x^p, with real number p)

  - Trig functions
  - Exponential and log functions
  - even and odd functions
- Inverse functions
  - One-to-one functions (passing the horizontal line test)
  - Properties of functions and their inverses (e.g. exchange of domains and ranges; mirror reflections, about y=x; intersections occur along the mirror line; slopes of tangent lines reflect in the mirror)
  - Compute the inverse to a given function, with domain restriction if necessary. Draw the inverse, given the graph of f.
  - Compute the derivative of the inverse of a function f using the general formula: $\frac{d\, f^{-1}[y]}{dy} == \frac{1}{f'[f^{-1}[y]]}$
- Indeterminate limits
  - L'Hopital's rule, which applies to an indeterminate quotient. No flat tire, no L'Hopital!
  - The different indeterminate forms, and transforming them to a quotient.
  - Asymptotics -- all rational functions look like polynomials in the limit, which one can discover by long division.
- Exponential functions
  - Exponential properties (e.g. exponential of a sum is the product of the exponentials)
  - The special properties which lead to the choice of base e^x.
  - Compute their limits, derivatives, and integrals ($\frac{d}{dx}\left(e^x\right) = e^x$, $\int e^x\, dx = e^x + C$)
  - Chain rule with exponentials: ($\frac{d}{dx}\left(e^{g(x)}\right)$ = g'(x) $e^{g(x)}$).
  - Hyperbolic trig functions, with interesting properties similar to those for trig functions.
- Logarithms
  - Invert exponential equations
  - Logarithm properties (which are reflections of the exponential properties in the mirror)
  - Compute their limits, derivatives, and integrals ($\frac{d}{dx}(\ln(x)) = \frac{1}{x}$, $\int \frac{1}{x}\, dx = \ln(|x|) + C$)
  - Chain rule with Logarithms: ($\frac{d}{dx}(\ln(g(x))) = \frac{g'(x)}{g(x)}$)
  - Logarithmic differentiation: e.g. to compute the derivative of x^x.
- Inverse trigonometric functions
  - Inverse sine, inverse cosine, and inverse tangent
  - Compute their limits, derivatives, and integrals ($\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$, $\frac{d}{dx}\left(\cos^{-1}(x)\right) = -\frac{1}{\sqrt{1-x^2}}$, $\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$)
  - shapes (e.g. sigmoidal arctan), limits, etc. Obtaining one from another via transformation (e.g. $\cos^{-1}(x)$ from $\sin^{-1}(x)$)
- Integration by parts (product rule backwards)

    - $\int u\,dv = u\,v - \int v\,du$, or my favorite $\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int f'(x)\,g(x)\,dx$
    - The idea that you're replacing one integral with another, and focused on a product for an integrand.
- Trigonometric integrals
    - Trigonometric identities (we only need these three, but others follow, mentioned below):
        - Pythagorean: $\cos^2(a\,x) + \sin^2(a\,x) = 1$.
        - cos(a + b) = cos(a)cos(b)-sin(a)sin(b)
        - sin(a + b) = sin(a)cos(b)+cos(a)sin(b)
    - $\int \sin^n(a\,x)\cos^m(a\,x)\,dx$ if at least one of n or m is an odd integer. Use substitution $u = \sin(a\,x)$, $du = a\cos(a\,x)\,dx$ or $u = \cos(a\,x)$, $du = -a\sin(a\,x)\,dx$, depending on whether n or m is odd. Then use Pythagorean identity.
    - $\int \sin^n(a\,x)\cos^m(a\,x)\,dx$ if both are even integers. Use trigonometric identities $\sin^2(a\,x) = \frac{1-\cos(2\,ax)}{2}$ and $\cos^2(a\,x) = \frac{1+\cos(2\,ax)}{2}$ to reduce the powers.
    - $\int \tan^n(a\,x)\sec^m(a\,x)\,dx$ if m is even. Use substitution $u = \tan(a\,x)$, $du = a\sec^2(a\,x)\,dx$ along with the Pythagorean identity $\sec^2(a\,x) = 1 + \tan^2(a\,x)$.
    - $\int \tan^n(a\,x)\sec^m(a\,x)\,dx$ if n is odd. Use substitution $u = \sec(a\,x)$, $du = a\sec(a\,x)\tan(a\,x)\,dx$ along with the Pythagorean identity $\tan^2(a\,x) = \sec^2(a\,x) - 1$
- Trigonometric substitution
    - If the integral involves $\sqrt{a^2 - x^2}$ use $x = a\sin(\theta)$ and $dx = a\cos(\theta)\,d\theta$.
    - If the integral involves $\sqrt{a^2 + x^2}$ use $x = a\tan(\theta)$ and $dx = a\sec^2(\theta)\,d\theta$.
    - If the integral involves $\sqrt{x^2 - a^2}$ use $x = a\sec(\theta)$ and $dx = a\sec(\theta)\tan(\theta)\,d\theta$.

Studying

- Try problems you haven't worked from the exercises from the corresponding sections of our textbook(s).
- Review the assignments. Remember which ones caused you the most trouble. Find similar examples in the textbook to try.
- Review the daily agendas. I review them before writing the exams....
- Contact me by email if you have questions. You can also visit the Math/Stats Tutoring Lab.