Exam 3: MAT 229, Spring 2025

Name:

Show your work to receive credit

A. Analyze each of the given series. If a series diverges, give reasons why; if it converges, either

- give the exact value it converges to, or
- provide an error estimate in approximating the sum with the 10th partial sum.

1. $\sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k$

2. $\sum_{k=1}^{\infty} \frac{k}{30k-20}$

3. $\sum_{k=1}^{\infty} \frac{2}{k^3}$

4. $\sum_{k=0}^{\infty} e^{-2k}$

5. $\sum_{k=0}^{\infty} (-1)^k \frac{3}{k+1}$

6. $\sum_{k=53}^{\infty} \frac{k}{k^2-1}$

B. Determine, by any of our methods, the partial sum S_n needed to assure that the error in computing an approximation to the infinite series S will be within 0.0001 of the correct value.

See how far you can get without resorting to a calculator. For example, if you get the problem down to say n > ln(2000), you're done!

7. $S_n = \sum_{k=0}^{n} \frac{1}{1+k^2+\cos^2(k)}$ (suggestion: use comparison)

8. $S_n = \sum_{k=0}^{n} (-1)^{k+1} \frac{1}{1+k^2}$

C. For $f(x) = e^{-2x}$:

9. Determine the 3rd Taylor polynomial T_3 for f about the center $a = 0$.

10. Use the remainder term R_3 to provide a bound on the error that might occur when approximating f by T_3 on the interval $\left[\frac{-1}{2}, \frac{1}{2}\right]$.