1. $\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k^2 4^k}$ $L = \lim_{k\to\infty} \left| \frac{\frac{1}{(k+1)^2 4^{k+1}}}{\frac{1}{4^k k^2}} \right| \rightarrow \lim_{k\to\infty} \left| \frac{1}{(k+1)^2 4^{k+1}} \cdot \frac{4^k \cdot k^2}{1} \right|$

$\downarrow$

$\lim_{k\to\infty} \left| \frac{k^2}{(k+1)^2 \cdot 4} \right|$

$\lim_{k\to\infty} \left| \frac{k^2}{4(k^2+2k+1)} \right| = \frac{1}{4} = L$

if $L<1$ then series converges: $\frac{1}{4} < 1$
-absolutely convergent

2. $\sum_{k=0}^{\infty} \frac{(-1)^{k-1}}{\sqrt{2k+3}}$ $L = \lim_{k\to\infty} \left| \frac{\frac{1}{\sqrt{2k+3}}}{\sqrt{2(k+1)+3}} \right| \rightarrow \lim_{k\to\infty} \left| \frac{\sqrt{2k+5}}{\sqrt{2k+3}} \right| = 1$

if $L=1$: inconclusive

Integral

$\int_0^{\infty} \frac{1}{|\sqrt{2k+3}|} dk = \int_0^{\infty} \frac{1}{2\sqrt{u}} du$ $\quad u = 2k+3$

$= \frac{1}{2} \cdot \int_0^{\infty} u^{-1/2} du$ $\quad du = 2\,dk$

$= \frac{1}{2} \frac{\sqrt{(2k+3)}}{1/2} \Big|_0^{\infty}$ $\quad \frac{1}{2} du = dk$

$= \sqrt{2k+3} \Big|_0^{\infty} = (\sqrt{2(\infty)+3}) - (\sqrt{2(0)+3}) = \infty$

This doesn't diverge absolutely

A.S.T $\lim_{k\to\infty} \left(\frac{1}{\sqrt{2k+3}} \right) = \frac{1}{\infty} = 0$

by A.S.T this is convergent,
so this is conditionally convergent

1.1 $\sum_{k=1}^{\infty} \frac{(-1)(k-1)}{(k)^2(4^k)}$ $L = \lim_{k\to\infty} \left| \frac{\frac{1}{(k+1)^2(4)^{n+1}}}{\frac{1}{(4)^k(k^2)}} \right|$

$\lim_{k\to\infty} \left| \frac{1}{(k+1)^2\ (4)^{k+1}} \cdot \frac{(4)^k \cdot (k)^2}{1} \right|$

$\lim_{k\to\infty} \left| \frac{k^2}{(k+1)^2(4)} \right|$

$\lim_{k\to\infty} \left| \frac{k^2}{4(k^2+2k+1)} \right| = \frac{1}{4}$ $L = \frac{1}{4}$

$L<1$ so the series converges, $\frac{1}{4}<1$

absolutely convergent

2. $\sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{\sqrt{2k+3}}$ $L = \lim_{k\to\infty} \left| \frac{\frac{1}{\sqrt{2k+3}}}{\sqrt{2(k+1)+3}} \right|$

$\lim_{k\to\infty} \frac{\sqrt{2k+3}}{\sqrt{2k+3}} = 1$

$\int_0^{\infty} \frac{1}{1\sqrt{2k+3})} dk = \int_0^{\infty} \frac{1}{2\sqrt{u}} du$

$= \int_0^{\infty} (u)^{-\frac{1}{2}} du$

$= \frac{1}{2} \cdot \frac{\sqrt{2k+3}}{\frac{1}{2}} \Big|_0^{\infty}$

$= \sqrt{2k+3} \Big|_0^{\infty}$

$= (\sqrt{2(\infty)+3}) - (\sqrt{2(0)+3}) = \infty$

$\lim_{k\to\infty} \left(\frac{1}{\sqrt{2k+3}} \right) = \frac{1}{\infty} = 0$ due to A.S.T this series in convergent

conditionally convergent

absol

2.1 $\sum_{n=0}^{\infty} \frac{(x+3)^n}{(n+1)3^{n+1}}$

The center is $\underline{\underline{-3}}$

2.2 $\sum_{n=0}^{\infty} \frac{(x+3)^n}{(n+1)3^{n+1}} = \frac{(x+3)^0}{(0+1)3^{0+1}} + \frac{(x+3)^1}{(1+1)3^{1+1}} + \frac{(x+3)^2}{(2+1)3^{2+1}} + \cdots$

$= \frac{1}{3} + \frac{x+3}{18} + \frac{(x+3)^2}{81} + \cdots$

$P(-3) = \frac{1}{3} - \frac{0}{18} + \frac{0}{81} + \cdots$

$= \underline{\underline{1/3}}$

2.3 Ratio test

$$\lim_{n\to\infty} \left| \frac{(x+3)^{\cancel{n+1}}}{((n+1)+1)\, 3^{\cancel{(n+1)}+1}} \cdot \frac{(n+1)\, \cancel{3^{n+1}}}{\cancel{(x+3)^n}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(n+1)(x+3)}{3(n+2)} \right|$$

$$\lim_{n\to\infty} \left| \frac{n+1}{3(n+2)} \right| |x+3|$$

$$= \frac{1}{3} |x+3| < 1$$

$$-1 < \frac{1}{3}(x+3) < 1$$

$$-3 < x+3 < 3$$

$$-6 < x < 0$$

Test Endpoints:

$$\sum_{n=0}^{\infty}\frac{(-6+3)^n}{(n+1)3^{n+1}} = \sum_{n=0}^{\infty}\frac{(-3)^n}{(n+1)3^{n+1}}$$

Converges by Alt. Series Test

$$\sum_{n=0}^{\infty}\frac{(0+3)^n}{(n+1)(3^{n+1})} = \sum_{n=0}^{\infty}\frac{3^n}{(n+1)3^{n+1}} = \sum_{n=0}^{\infty}\frac{1}{3(n+1)}$$

$$\int_0^{\infty}\frac{1}{3x+3}\,dx = \frac{1}{3}\ln(x+1)$$

Diverges

Interval of Convergence: $[-6, 0)$

Good work

3.) 3.1) $f(x) = \ln(1+x) \Rightarrow f'(x) = (1+x)^{-1}$

3.2) $\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty}(-x)^n = 1 + (-x) + x^2 - x^3 + x^4 \ldots$

3.3) $\int \sum_{n=0}^{\infty}(-x)^n\,dx = \int(1 - x + x^2 - x^3 + x^4 \ldots)\,dx = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}\ldots$

$= \sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$

good

3.4) $\lim_{n\to\infty}\left|\frac{x^{n+1}}{x^n}\right| < 1 \Rightarrow |x| < 1 \Rightarrow \begin{matrix}\ln(1-1) = -\infty \\ \ln(1+1) = \ln(2)\end{matrix} \Rightarrow \boxed{(0, 2]}$

radius of convergence?

3.1) $f(x) = \ln(1+x)$

$$f'(x) = \frac{1}{1+x}$$

3.2) $f'(x) = \frac{1}{1+x}$

$$f'(x) = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n$$

$$\boxed{= \sum_{n=0}^{\infty} (-1)^n x^n}$$

3.3) $f(x) = \int \sum_{n=0}^{\infty} (-1) x^n \, dx = \int (1 - x + x^2 - x^3 + x^4 - x^5 + \ldots) \, dx$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \ldots + C$$

$$f(x) = \int \sum_{n=0}^{\infty} (-1) x^n dx = \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}}$$

3.4) $\lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{n+1}}{(-1)^n x^n} \right| = \lim_{n\to\infty} \left| \frac{x^{n+1}}{x^n} \right|$

$$= \lim_{n\to\infty} |x|$$

$$= |x| < 1$$

$$-1 < x < 1$$

Radius of convergence for $\sum_{n=0}^{\infty} (-1)^n x^n$ is 1

$\lim_{n\to\infty} \left| \frac{x^{(n+1)+1}}{(n+1)+1} \Big/ \frac{x^{n+1}}{n+1} \right| = \lim_{n\to\infty} \left| \frac{x^{n+2}}{n+2} \cdot \frac{n+1}{x^{n+1}} \right|$

$$= \lim_{n\to\infty} \left| \frac{n+1}{n+2} \cdot x \right|$$

$$= |x| < 1$$

$$-1 < x < 1$$

Radius of convergence for $\sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$ is 1

4. $g(x) = -\sqrt{1+x^2}$

$-\sqrt{32} = -\sqrt{16 \cdot 2}$
$= 4\sqrt{2}$

4.1) $g(1) = -\sqrt{1+1^2} = -\sqrt{2}$

$g'(1) = -\frac{1}{\sqrt{1+1^2}} = \frac{1}{\sqrt{2}}$

$g''(1) = \frac{1}{(1+1^2)^{3/2}} = \frac{1}{2\sqrt{2}}$

$g'''(1) = -\frac{3(1)}{(1+1^2)^{5/2}} = \frac{-3}{\sqrt{32}}$

$T_4(x) = -\sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{\frac{1}{2\sqrt{2}}}{2!}(x-1)^2 - \frac{\frac{3}{\sqrt{32}}}{3!}(x-1)^3$

$\boxed{= -\sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{4\sqrt{2}}(x-1)^2 - \frac{1}{8\sqrt{2}}(x-1)^3}$

4.2) $|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1}, \quad M \ge |f^{(n+1)}(x)|$

$M \ge |f^4(x)|$ from $0 \le x \le 2$

$M \ge |-3|$

$\boxed{M \ge 3}$

error bound: $|R_3(x)| \le \frac{3}{4!}|x-1|^4$

$\downarrow$

$\boxed{|R_3(x)| \le \frac{1}{8}|x-1|^4}$