Practice Final Exam

MAT 229, Spring 2021

1. Consider the function $y(x)$ defined implicitly by $x = \log_2(y)$.

1.1. Compute the derivative dy/dx of the function $y(x)$.

1.2. Find the tangent line to the graph of $y(x)$ at the point (1,2).

2. Compute the integral using integration by parts, showing all the steps in your work.

$$\int \tan^{-1}(2x)\,dx$$

3. Evaluate $\int \cos^3(2x)\sin^3(2x)\,dx$ using an appropriate trig substitution.

4. (Power series) Consider the function given as the power series centered at 0.

$$f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+2}}{(n+1)\,3^n}$$

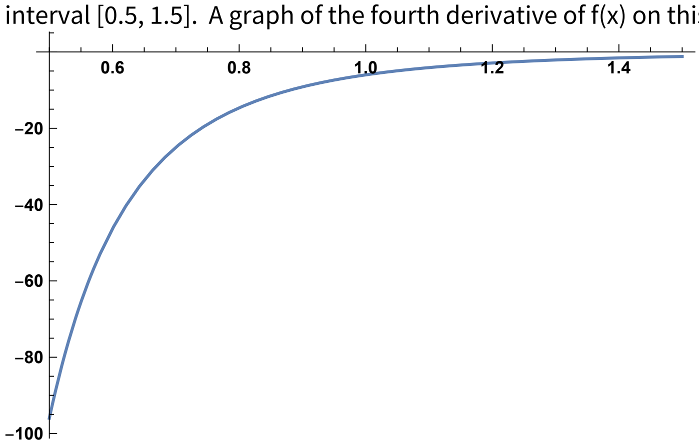
4.1. What is the domain for $f(x)$? In other words, what is the interval of convergence?

4.2. Find a power series centered at 0 for $f'(x)$ and give its radius of convergence.

5. We wish to approximate the function $f(x) = \ln(x)$ as a Taylor polynomial of degree 3, $T_3(x)$, in the vicinity of $x = 1$.

5.1. Write the polynomial.

5.2. Use the Taylor remainder theorem to give a good bound on the error of our approximation on the interval [0.5, 1.5]. A graph of the fourth derivative of f(x) on this interval is provided here:



6. Consider the integral $\int_0^2 \frac{1}{1+x}\,dx$.

6.1. Approximate it using the midpoint rule with 2 rectangles.

6.2. What does the midpoint rule error estimate give as the maximum error in this approximation?

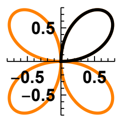
7. Two of the following three series can be evaluated almost instantaneously; the third is divergent. Give their values, or prove divergence.

7.1. $\sum_{n=0}^{\infty} \frac{2^{n+1}}{(-3)^n}$

7.2. $\sum_{n=0}^{\infty} \frac{2^n}{n!}$

7.3. $\sum_{n=0}^{\infty} \frac{1+\frac{1}{n}}{(n+1)}$

8. Determine whether the improper integral $\int_e^{\infty} \frac{1}{x \ln(x)} dx$ converges or not. Give reasons for your answer.

9. This is the graph 

of the beautiful "rose function", $r = \sin(2\,\theta)$.

9.1. What is the period of this function?

9.2. Give a choice of angle values θ that trace out just the dark petal.

9.3. Write an integral representing the area of one petal, and compute that area (including a decimal value).

10. The curve given parametrically by $x = \cos(2\,t)$, $y = \sin(3\,t)$ is shown below. It has two points with horizontal tangents. Determine the coordinates (x, y) for those two points.

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ParametricPlot[{Cos[2 t], Sin[3 t]}, {t, 0, 4 π},
 Ticks → None, AxesLabel → {x, y}, BaseStyle → FontSize → 14]
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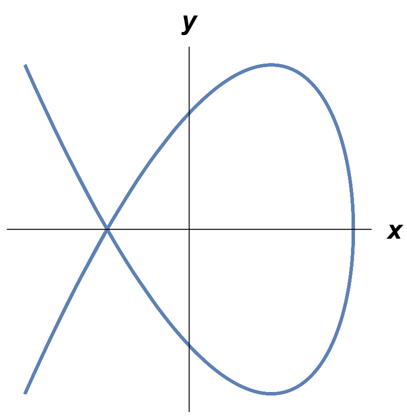

11. Consider the vector $\vec{v} = \langle -1, 3, 2 \rangle$

11.1. Compute the exact length of $\vec{v}$ (that is, the norm of $\vec{v}$, or $|\vec{v}|$).

11.2. Find a vector $\vec{u}$ perpendicular to $\vec{v}$ (and **show** that it is perpendicular).

11.3. Use the cross product to find a vector perpendicular to both $\vec{u}$ and $\vec{v}$.

12. Consider the plane through the three points $P_0(1, 0, 0)$, $P_1(1, 1, 0)$, $P_2(0, 1, 1)$. Find an equation for this plane.