Section Summary: 11.7

1 Some suggestions for testing for convergence

a. If the series is of the form

$$\sum \frac{1}{n^p}$$

then it's a p-series, converging for $p > 1$ and diverging otherwise.

b. If the series is of the form

$$\sum cr^n$$

then it's a geometric series, converging for $|r| < 1$ and diverging otherwise.

c. If a series is similar to either of the series above, then it may be possible to use comparison with those series. Comparison may involve a judicious throwing away of stuff! It's only necessary that series behave *eventually* – their "heads" can do whatever, because it's the "tail" of the series that determines convergence.

d. If

$$\lim_{n\to\infty} a_n \neq 0$$

then the series diverges by the Test for Divergence.

e. If the series is of the form

$$\sum (-1)^{n-1} b_n$$

then the Alternating Series Test is a good bet.

f. Series that involve factorials or other products may be best approached with the Ratio Test.

g. If a_n is of the form $(b_n)^n$, then the Root Test is a good candidate.

h. If $a_n = f(n)$, where $\int_1^\infty f(x)dx$ is easily evaluated, then the Integral Test may be effective.

i. Absolute convergence implies convergence, so if it's easier to demonstrate that the series

$$\sum |a_n|$$

is convergent, then show that.

2 Summary

These are just some suggestions that might prove helpful as you examine the convergence of a series. There is no magic bullet. The good news is that actually **several different techniques** may work.

Remember that the reason we're concerned about convergence is because we'd like to know the limit. If the series is convergent, then we can use partial sums to **approximate** the limit (even if we can't compute it exactly). And in many cases, we can calculate the limit to a given tolerance – we can give an approximation that's as exact as you'd like.