Lab 1: Student Assignment

Week 1, January 11-15

MAT 229, Spring 2021

Lab 1 key: Andy Long

Special Constants

Standard notation	Mathematica notation
$\pi \approx 3.14159$	**Pi**
$e \approx 2.71828$	**E**

Commands

Functionality	Mathematica notation
plot the graph of a function	**Plot[...]**
square root of something, $\sqrt{\ldots}$	**Sqrt[...]**
absolute value of something, $\lvert \ldots \rvert$	**Abs[...]**
sine of something (radian mode), sin(…)	**Sin[...]**
cosine of something (radian mode), cos(…)	**Cos[...]**
tangent of something (radian mode), tan(…)	**Tan[...]**

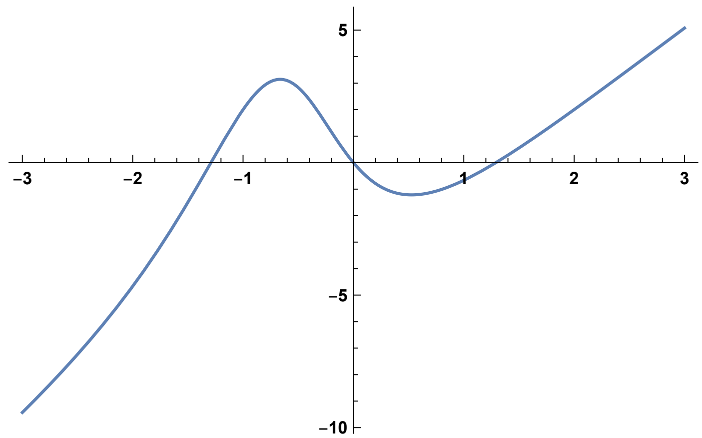
Exercises to submit

Exercise 1

Plot the graph of $\frac{3x^3-5x}{x^2+x+1}$ for $-3 \le x \le 3$.

```
In[ ]:= Plot[(3 x^3 - 5 x) / (x^2 + x + 1), {x, -3, 3}]
```

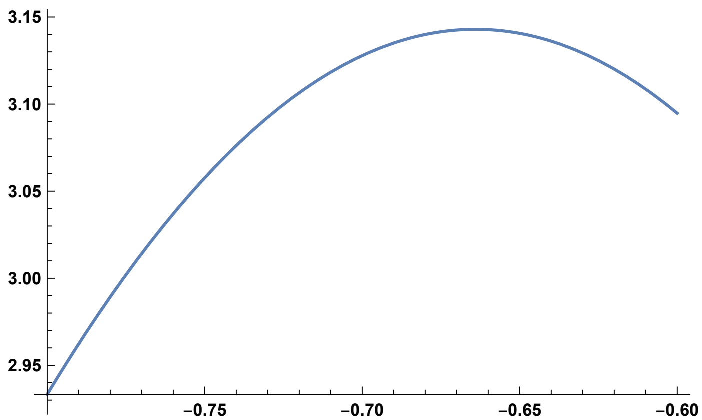
Out[•]=



Zooming in is a good strategy. Estimation is a really important skill, and you want to develop your skills with estimation.

In[•]:=
```
Plot[(3 x^3 - 5 x) / (x^2 + x + 1), {x, -0.8, -0.6}]
```

Out[•]=

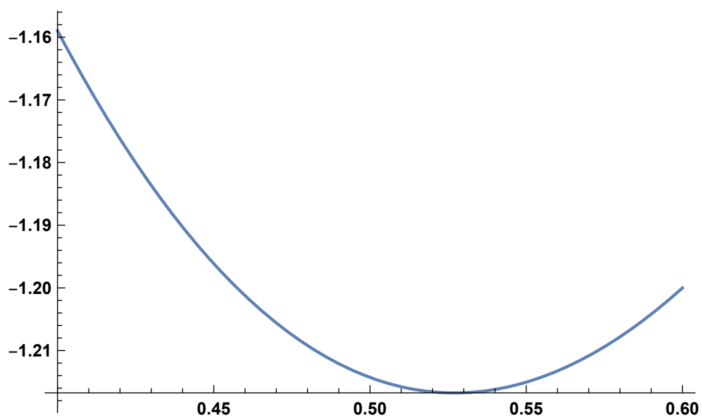


- From the graph estimate the values of *x* for which this graph has a local maximum point. Try to get 1 decimal place of accuracy.

  Local maximum points: $x \approx$ -0.7 (closer to that than to -0.6)

In[•]:=
```
Plot[(3 x^3 - 5 x) / (x^2 + x + 1), {x, 0.4, 0.6}]
```

Out[•]=



- From the graph estimate the values of x for which this graph has a local minimum point. Try to get 1 decimal place of accuracy.

  Local minimum points: $x \approx$ 0.5 (closer to that than to 0.6).

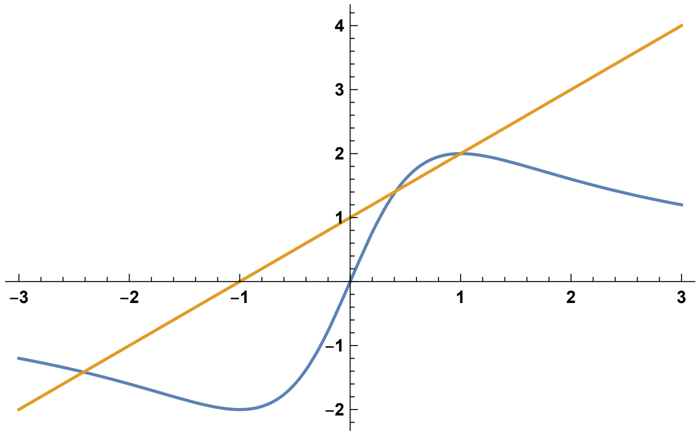
Exercise 2

Let $g(x) = \frac{4x}{1+x^2}$ and $h(x) = x + 1$.

- Plot the graphs of $g(x)$ and of $h(x)$ together on the same axes for $-3 \leq x \leq 3$.

In[•]:=
```
Plot[{4 x / (1 + x^2), x + 1}, {x, -3, 3}]
```

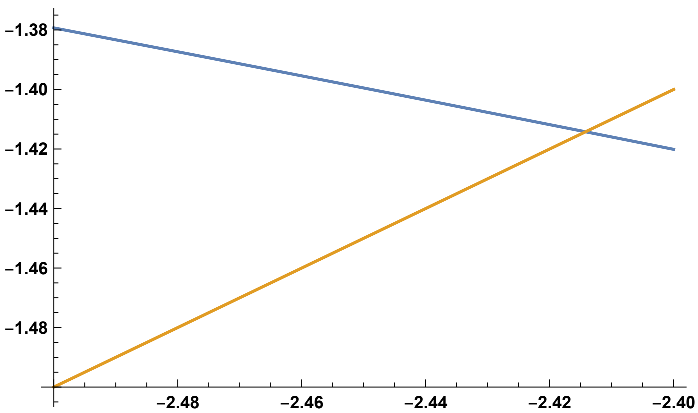
Out[•]=



Zoom in again, this time on three points:

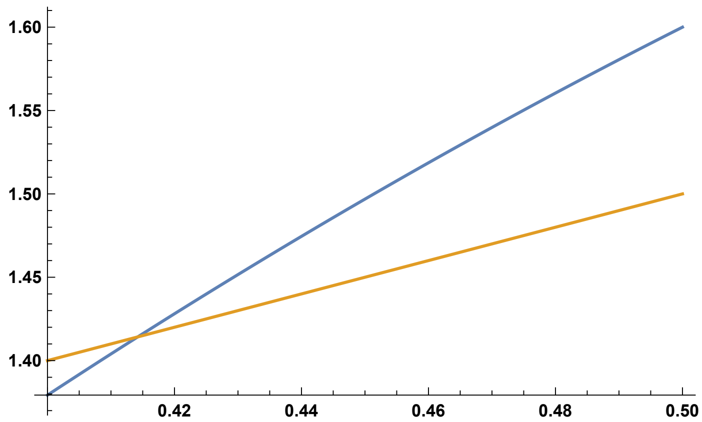
In[•]:=
```
Plot[{4 x / (1 + x^2), x + 1}, {x, -2.5, -2.4}]
```

Out[•]=



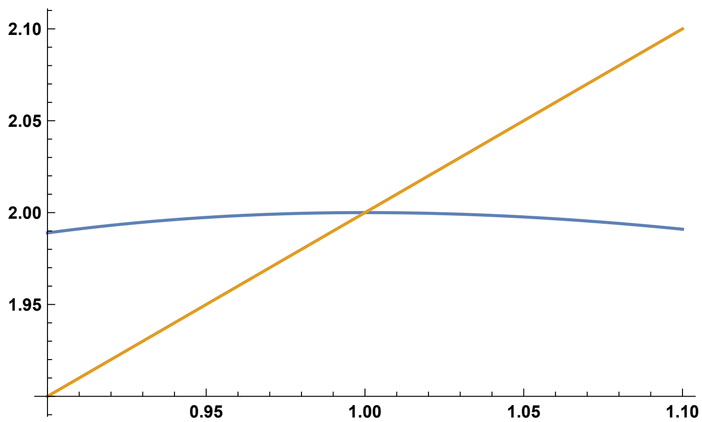
In[•]:=
```
Plot[{4 x / (1 + x^2), x + 1}, {x, 0.4, 0.5}]
```

Out[•]=



In[•]:=
```
Plot[{4 x / (1 + x^2), x + 1}, {x, 0.9, 1.1}]
```

Out[•]=



- From your plot estimate the values of *x* where these two graphs intersect. Try to get 1 decimal place of accuracy.

  Intersection points: $x \approx$ {-2.4, 0.4, 1} (the last one is exact, as one can check using the formulas).

In[177]:=
```
Solve[4 x / (1 + x^2) == x + 1, x]
```

Out[177]= $\{\{x \to 1\}, \{x \to -1 - \sqrt{2}\}, \{x \to -1 + \sqrt{2}\}\}$

Exercise 3

Let $f(x) = \sin(x)$.

Use point slope form: y-y0 = f'(x0)(x-x0), or y = y0 + f'(x0)(x-x0)
f'(x)=cos(x); f'(0)=1; y0=f(x0), so y0=sin(0)=0; hence

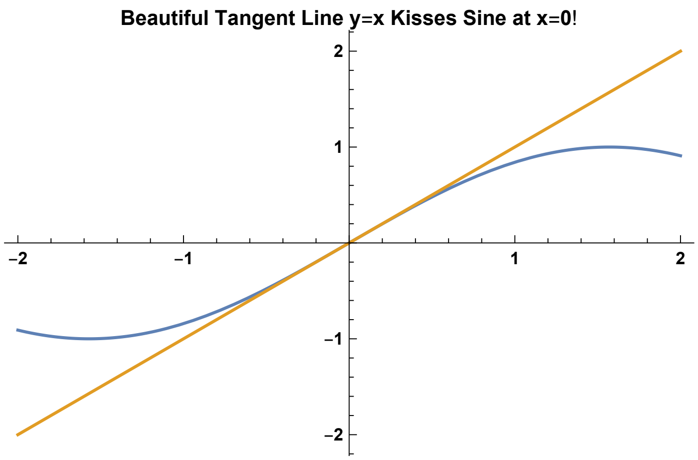
- Determine an equation for the tangent line to $f(x) = \sin(x)$ at $x = 0$, $y = m x + b$.

  $y = x$

- Plot graphs of $f(x)$ and your line $m x + b$ on the same graph for $-2 \leq x \leq 2$.

In[•]:=
```
Plot[{Sin[x], x}, {x, -2, 2},
 PlotLabel → "Beautiful Tangent Line y=x Kisses Sine at x=0!"]
```

Out[•]=



Exercise 4

The linear approximation for a function $f(x)$ at $x = a$ is the function $L(x) = m\,x + b$ that comes from the tangent line $y = m\,x + b$ to $f(x)$ at $x = a$. It provides a simple approximation to $f(x)$ for values of x near a:

$$f(x) \approx L(x) = m\,x + b$$

Let $f(x) = \tan(2\,x)$.

- Determine the linear approximation for $f(x)$ at $x = 0$.

  $\tan(2\,x) \approx m\,x + b =$ 2x

This is equivalent to asking for the tangent line to the graph of tan(2x) at x=0: so
y = y0 + f'(x0)(x-x0).
f'(x)=2/(cos(2x))^2, so f'(0) = 2; y0=tan(0)=0; hence y = 2x is the tangent line.

In[•]:=
```
Plot[{Tan[2 x], 2 x}, {x, -.5, .5}]
```

- Using your values for m and b, plot the absolute value of the difference of this function and its linear approximation

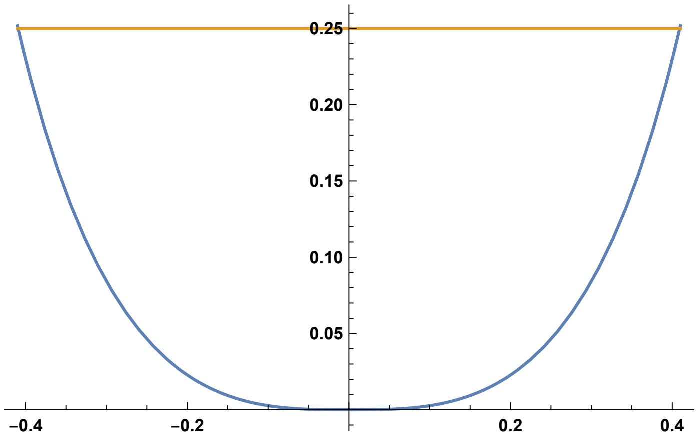
  $$|\tan(2\,x) - (m\,x + b)|$$

  (Note: Use `Abs[...]` for the absolute value)
  By varying the values of x for which you plot this difference, estimate the x-values for which this difference is no more than 0.25.

  $-0.41 \le x \le 0.41$ (it's symmetric, in any event)

In[•]:=
```
Plot[{Abs[Tan[2 x] - 2 x], 0.25}, {x, -.41, .41}]
```

Out[ ]=



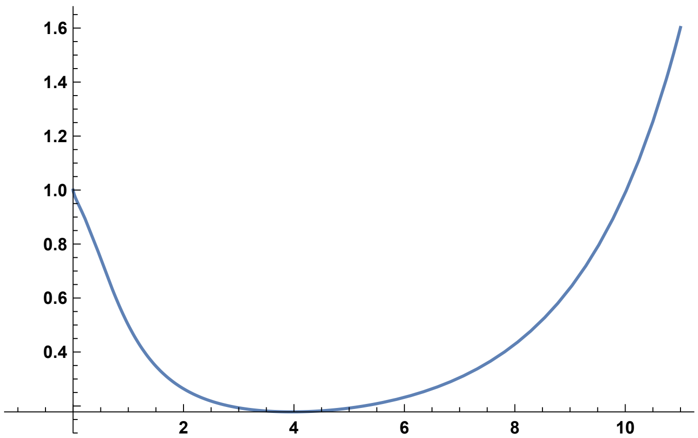
Exercise 5

Consider the function $f(x) = \frac{x^{(x/5)}}{1+x^2}$

- Plot it for various ranges of *x*-values until you get one that shows all the important aspects of the graph.

In[ ]:=
```
Plot[x^(x/5)/(1+x^2), {x, -1, 11}]
```

Out[ ]=



- From the graph, what is the domain of this function?

  Domain: (0, Infinity)

Notice that it's open at 0. Remember to check those boundaries in Mathematica! What does Mathematica say when you type 0^0?

In[172]:=
```
0^0
f[x_] := x^(x/5)/(1+x^2)
f[0]
```

Out[172]= Indeterminate

Out[174]= Indeterminate

- From the graph, what are the intervals of decrease?
  Intervals of decrease: (0, 3.96)
- From the graph, what are the intervals of increase?
  Intervals of increase: (3.96, Infinity)

Let's just zoom in there to verify that the minimum is at around x=3.96:

In[ ]:=
```
Plot[x^(x/5)/(1+x^2), {x, 3.9, 4.0}]
```

Out[ ]=