Lab 10: Instructors’ notes

Week 10

MAT 229, Spring 2021

Sequence

Functions

If you have a formula for a sequence, say $a_n = (-1)^n \frac{2^n}{n!}$, you can define it as a function as usual.

In[1048]:=
```
Clear[n]
a[n_] := (-1)^n 2^n/n!
```

In[1050]:=
```
a[n]
```

Out[1050]= $\frac{(-2)^n}{n!}$

Be aware that often you can only evaluate it for whole number values of *n*.

Legitimate input values

In[1051]:=
```
a[2]
```

Out[1051]= 2

In[1052]:=
```
a[3]
```

Out[1052]= $-\frac{4}{3}$

Non-legitimate input values

In[1053]:=
```
a[1/2]
```

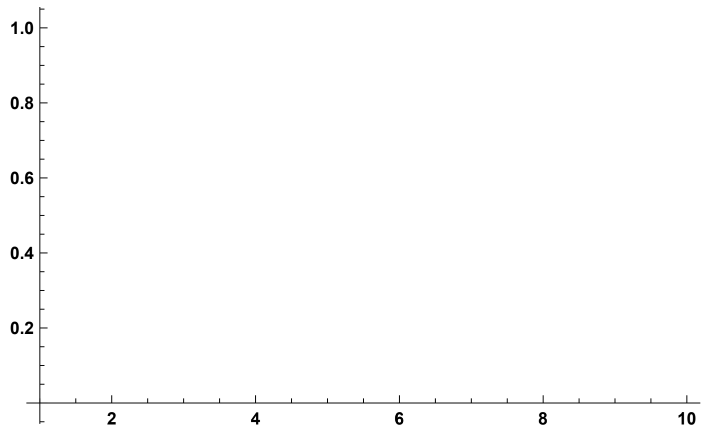
Out[1053]= $2\,i\sqrt{\frac{2}{\pi}}$

In[1054]:=
```
a[-3]
```

Out[1054]= 0

In[1087]:=
```
Plot[a[n], {n, 1, 10}]
a[2.2]
```

Out[1087]=



Out[1088]= 1.53355 + 1.11419 ⅈ

Generating values

To generate the first few values of a sequence, use the Table command.

In[1079]:=
```
Table[a[k], {k, 1, 10}]
```

Out[1079]= $\left\{-2, 2, -\frac{4}{3}, \frac{2}{3}, -\frac{4}{15}, \frac{4}{45}, -\frac{8}{315}, \frac{2}{315}, -\frac{4}{2835}, \frac{4}{14175}\right\}$

In[1057]:=
```
Table[N[a[k]], {k, 1, 10}]
```

Out[1057]= {-2., 2., -1.33333, 0.666667, -0.266667,
0.0888889, -0.0253968, 0.00634921, -0.00141093, 0.000282187}

Note

The name of the index does not matter.

In[1058]:=
```
Table[N[a[n]], {n, 1, 10}]
```

Out[1058]= {-2., 2., -1.33333, 0.666667, -0.266667,
0.0888889, -0.0253968, 0.00634921, -0.00141093, 0.000282187}

Try

Create the first 30 decimal values for the sequence $\left\{(-1)^{j+1}\,\frac{1}{2j-1}\right\}_{j=1}^{\infty}$.

In[1059]:=
```
Table[(-1)^(j+1) 1./(2 j - 1), {j, 1, 30}]
```

Out[1059]=
```
{1., -0.333333, 0.2, -0.142857, 0.111111, -0.0909091,
 0.0769231, -0.0666667, 0.0588235, -0.0526316, 0.047619, -0.0434783,
 0.04, -0.037037, 0.0344828, -0.0322581, 0.030303, -0.0285714,
 0.027027, -0.025641, 0.0243902, -0.0232558, 0.0222222, -0.0212766,
 0.0204082, -0.0196078, 0.0188679, -0.0181818, 0.0175439, -0.0169492}
```

Try

To see the index along with the value of the sequence at the location include {n, a_n} in the Table. You can give it a name as well.

In[1060]:=
```
sequence = Table[{j, (-1)^(j+1) 1./(2 j - 1)}, {j, 1, 30}]
```

Out[1060]=
```
{{1, 1.}, {2, -0.333333}, {3, 0.2}, {4, -0.142857}, {5, 0.111111},
 {6, -0.0909091}, {7, 0.0769231}, {8, -0.0666667}, {9, 0.0588235},
 {10, -0.0526316}, {11, 0.047619}, {12, -0.0434783}, {13, 0.04}, {14, -0.037037},
 {15, 0.0344828}, {16, -0.0322581}, {17, 0.030303}, {18, -0.0285714},
 {19, 0.027027}, {20, -0.025641}, {21, 0.0243902}, {22, -0.0232558},
 {23, 0.0222222}, {24, -0.0212766}, {25, 0.0204082}, {26, -0.0196078},
 {27, 0.0188679}, {28, -0.0181818}, {29, 0.0175439}, {30, -0.0169492}}
```

To view it as a table of values use TableForm or MatrixForm.

In[1061]:= TableForm[sequence]

Out[1061]//TableForm=

1	1.
2	-0.333333
3	0.2
4	-0.142857
5	0.111111
6	-0.0909091
7	0.0769231
8	-0.0666667
9	0.0588235
10	-0.0526316
11	0.047619
12	-0.0434783
13	0.04
14	-0.037037
15	0.0344828
16	-0.0322581
17	0.030303
18	-0.0285714
19	0.027027
20	-0.025641
21	0.0243902
22	-0.0232558
23	0.0222222
24	-0.0212766
25	0.0204082
26	-0.0196078
27	0.0188679
28	-0.0181818
29	0.0175439
30	-0.0169492

In[1062]:=
```
MatrixForm[sequence]
```

Out[1062]//MatrixForm=

1	1.
2	-0.333333
3	0.2
4	-0.142857
5	0.111111
6	-0.0909091
7	0.0769231
8	-0.0666667
9	0.0588235
10	-0.0526316
11	0.047619
12	-0.0434783
13	0.04
14	-0.037037
15	0.0344828
16	-0.0322581
17	0.030303
18	-0.0285714
19	0.027027
20	-0.025641
21	0.0243902
22	-0.0232558
23	0.0222222
24	-0.0212766
25	0.0204082
26	-0.0196078
27	0.0188679
28	-0.0181818
29	0.0175439
30	-0.0169492

Plotting sequences

Create and name a list of the first 100 decimal values for the sequence $\left\{100^{n} \frac{n!+(2n)!}{(3n)!}\right\}$.

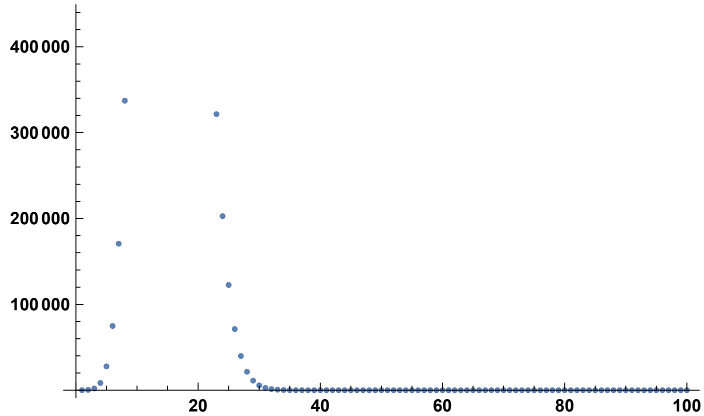
```
In[1063]:= list = Table[N[{n, 100^n (n! + (2 n)!) / (3 n)!}], {n, 1, 100}]

Out[1063]= {{1., 50.}, {2., 361.111}, {3., 2000.66}, {4., 8422.52}, {5., 27750.9},
  {6., 74816.4}, {7., 170634.}, {8., 337220.}, {9., 587974.}, {10., 917201.},
  {11., 1.29444×10^6}, {12., 1.6679×10^6}, {13., 1.97712×10^6}, {14., 2.17001×10^6},
  {15., 2.21742×10^6}, {16., 2.11964×10^6}, {17., 1.90335×10^6}, {18., 1.61145×10^6},
  {19., 1.29056×10^6}, {20., 980551.}, {21., 708666.}, {22., 488343.},
  {23., 321561.}, {24., 202728.}, {25., 122592.}, {26., 71225.9}, {27., 39820.4},
  {28., 21452.8}, {29., 11152.}, {30., 5600.66}, {31., 2720.5}, {32., 1279.52},
  {33., 583.271}, {34., 257.948}, {35., 110.769}, {36., 46.2272}, {37., 18.7633},
  {38., 7.41282}, {39., 2.85251}, {40., 1.06988}, {41., 0.391365}, {42., 0.139713},
  {43., 0.0487025}, {44., 0.0165869}, {45., 0.00552214}, {46., 0.00179803},
  {47., 0.000572858}, {48., 0.000178671}, {49., 0.0000545776}, {50., 0.0000163347},
  {51., 4.79203×10^-6}, {52., 1.37852×10^-6}, {53., 3.89005×10^-7}, {54., 1.07721×10^-7},
  {55., 2.92823×10^-8}, {56., 7.8165×10^-9}, {57., 2.04958×10^-9}, {58., 5.28075×10^-10},
  {59., 1.33733×10^-10}, {60., 3.32983×10^-11}, {61., 8.15392×10^-12},
  {62., 1.96422×10^-12}, {63., 4.65597×10^-13}, {64., 1.08626×10^-13},
  {65., 2.49503×10^-14}, {66., 5.6433×10^-15}, {67., 1.25722×10^-15},
  {68., 2.75934×10^-16}, {69., 5.96779×10^-17}, {70., 1.27212×10^-17},
  {71., 2.67323×10^-18}, {72., 5.53898×10^-19}, {73., 1.13185×10^-19},
  {74., 2.2814×10^-20}, {75., 4.53673×10^-21}, {76., 8.90214×10^-22},
  {77., 1.72398×10^-22}, {78., 3.29555×10^-23}, {79., 6.21952×10^-24},
  {80., 1.15901×10^-24}, {81., 2.13299×10^-25}, {82., 3.87731×10^-26},
  {83., 6.96264×10^-27}, {84., 1.23534×10^-27}, {85., 2.16584×10^-28},
  {86., 3.75283×10^-29}, {87., 6.42748×10^-30}, {88., 1.08825×10^-30},
  {89., 1.82173×10^-31}, {90., 3.0155×10^-32}, {91., 4.93638×10^-33},
  {92., 7.99254×10^-34}, {93., 1.28009×10^-34}, {94., 2.02828×10^-35},
  {95., 3.17975×10^-36}, {96., 4.93272×10^-37}, {97., 7.57281×10^-38},
  {98., 1.15067×10^-38}, {99., 1.73066×10^-39}, {100., 2.57683×10^-40}}
```

Use the semicolon to suppress the output if it becomes too long.

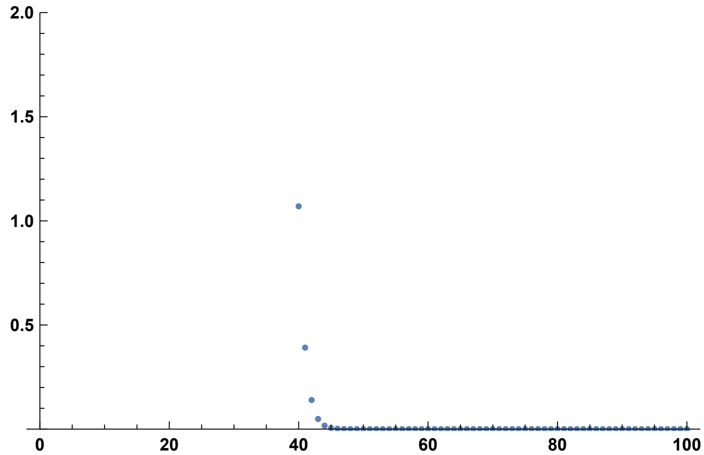
Use ListPlot to get a visualization a sequence

In[1064]:=
```
ListPlot[list]
```

Out[1064]=


Use the PlotRange option if you want to specify the *y*-range to view.

In[1065]:=
```
ListPlot[list, PlotRange → {0, 2}]
```

Out[1065]=


Series

Approximate infinite series with its partial sums.

Geometric series

$$\sum_{k=0}^{\infty}(0.7)^k \approx \sum_{k=0}^{10}(0.7)^k = 1 + 0.7 + (0.7)^2 + (0.7)^3 + \ldots + (0.7)^{10}$$

In[1066]:=
```
Sum[0.7^k, {k, 0, 10}]
```

Out[1066]= 3.26742

Note

Geometric series $\sum_{k=0}^{\infty} a\, r^k$ converges if $|r| < 1$ and converges to $a\,\frac{1}{1-r}$. What is the error in the approximation $\sum_{k=0}^{\infty}(0.7)^k \approx \sum_{k=0}^{10}(0.7)^k$?

Example

To study the series $\sum_{k=1}^{\infty}\frac{1}{k^3+1}$ create a sequence of partial sums $s_n = \sum_{k=1}^{n}\frac{1}{k^3+1}$.

```
Clear[n]
s[n_] := Sum[1/(k^3+1), {k, 1, n}]
```

```
Out[1101]= 1/3 (RootSum[1 + #1^3 &, PolyGamma[0, 1 - #1] #1 &] -
    RootSum[1 + #1^3 &, PolyGamma[0, 1 + n - #1] #1 &])
```

```
In[1068]:= s[1]
Out[1068]= 1/2
```

```
In[1069]:= s[10]
Out[1069]= 194 732 314 259 / 285 539 637 240
```

The exact value is not too helpful. Change the 1 to 1.0 to get just decimal values.

```
In[1102]:= s[n_] := Sum[1.0/(k^3+1), {k, 1, n}]
```

```
In[1090]:= s[10]
Out[1090]= 0.68198
```

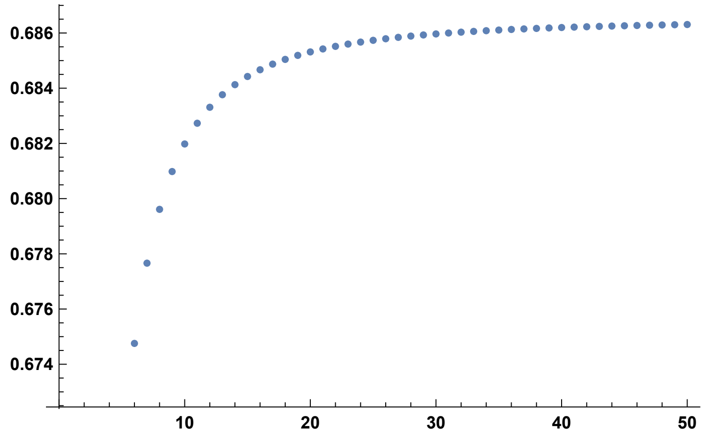
A sequence of the first 20 partial sums:

```
In[1104]:= partial = Table[{n, s[n]}, {n, 1, 50}]
Out[1104]= {{1, 0.5}, {2, 0.611111}, {3, 0.646825}, {4, 0.66221}, {5, 0.670147},
  {6, 0.674755}, {7, 0.677662}, {8, 0.679611}, {9, 0.680981}, {10, 0.68198},
  {11, 0.682731}, {12, 0.683309}, {13, 0.683764}, {14, 0.684128}, {15, 0.684425},
  {16, 0.684669}, {17, 0.684872}, {18, 0.685044}, {19, 0.685189}, {20, 0.685314},
  {21, 0.685422}, {22, 0.685516}, {23, 0.685598}, {24, 0.685671}, {25, 0.685735},
  {26, 0.685792}, {27, 0.685842}, {28, 0.685888}, {29, 0.685929}, {30, 0.685966},
  {31, 0.686}, {32, 0.68603}, {33, 0.686058}, {34, 0.686083}, {35, 0.686107},
  {36, 0.686128}, {37, 0.686148}, {38, 0.686166}, {39, 0.686183}, {40, 0.686199},
  {41, 0.686213}, {42, 0.686227}, {43, 0.686239}, {44, 0.686251}, {45, 0.686262},
  {46, 0.686272}, {47, 0.686282}, {48, 0.686291}, {49, 0.686299}, {50, 0.686307}}
```

Plot them.

In[1105]:=
```
ListPlot[partial]
```

Out[1105]=



Combine this plot with a plot of the horizontal line $y = 0.069$ using the Show command.

In[1107]:=
```
Show[
 Plot[0.69, {x, 1, 50}, PlotLabel → "Partial Sums"],
 (* Show takes its marching orders from the first plot.... *)
 ListPlot[partial]
]
```

Out[1107]=