Lab 10

MAT 229, Spring 2021

Exercises to submit

Exercise 1

Consider the sequence given by the formula $\{a_k\}_{k=1}^{\infty} = \left\{\frac{(k!)^2}{(2\,k)!}\right\}_{k=1}^{\infty}$. (The parentheses are important here. Make sure you have them in the correct places.)

a. Plot the first 20 terms of this sequence. From this plot, what do you think is the value of $\lim_{k\to\infty} \frac{(k!)^2}{(2\,k)!}$?

b. Plot the first 20 partial sums $S_n = \sum_{k=1}^{n} \frac{(k!)^2}{(2\,k)!}$. Based on this plot, do you think the series $\sum_{k=1}^{\infty} \frac{(k!)^2}{(2\,k)!}$ converges or not. If you think it converges, estimate its value.

Exercise 2

Consider the harmonic series $\sum_{k=1}^{\infty} \frac{1}{k}$ and the alternating harmonic series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$.

a. The harmonic series $\sum_{k=1}^{\infty} \frac{1}{k}$ diverges.

- Compute the partial sums for this series as **decimal values:**
  $S_{10}, S_{100}, S_{1000}, S_{10\,000}, S_{100\,000}, S_{1\,000\,000}$.
- Estimate by how much the partial sums go up when we increase the number of terms in the partial sum by a factor of 10.

b. The alternating harmonic series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$ converges.

- Compute the partial sums for this series as **decimal values:**
  $S_{10}, S_{100}, S_{1000}, S_{10\,000}, S_{100\,000}, S_{1\,000\,000}$.
- To 4 decimal places, what does the alternating harmonic series converge to?

Exercise 3

Geometric series have the form $\sum_{k=0}^{\infty} a\, r^k$. It converges if and only if $|r| < 1$. If it does converge it converges to $\frac{a}{1-r}$.

a. The geometric series $\sum_{k=0}^{\infty} (0.5)^k$ converges to $\frac{1}{1-0.5} = 2$. Create a plot of the partial sums of this series. Use it to help you zero in on the first partial sum that is within 0.001 of the series value.

- Plot the partial sums as points until you see where they level off.
- Use whatever means you have to zoom in around that leveling off to determine where the partial sums start getting within 0.0001 of the convergent value.

b. The geometric series $\sum_{k=0}^{\infty} (0.95)^k$ converges to $\frac{1}{1-0.95} = 20$. Determine the first partial sum that is within 0.0001 of the series value.

- Plot the partial sums as points until you see where they level off.
- Use whatever means you have to zoom in around that leveling off to determine where the partial sums start getting within 0.0001 of the convergent value.

Exercise 4

Let the sequence $\{a_k\}_{k=1}^{\infty}$ be given by $\left\{(-1)^{k(k+1)/2} \frac{k!}{10^k}\right\}_{k=1}^{\infty}$.

a. Create a list of the first 10 terms of this sequence.

- What is the pattern with the signs?
- Do you have a guess on what is the value of $\lim_{k\to\infty} a_k$?

b. Create a list of the first 20 terms of this sequence.

- What is the pattern with the signs?
- Do you have a guess on what is the value of $\lim_{k\to\infty} a_k$?

c. Create a list of the first 30 terms of this sequence.

- What is the pattern with the signs?
- Do you have a guess on what is the value of $\lim_{k\to\infty} a_k$?

d. What is true about $\sum_{k=1}^{\infty} a_k$? Give reasons for your answer.