Lab 12

MAT 229, Spring 2021

Today's lab is primarily about the ratio and the root tests for the convergence of a series.

Your job: to explore these examples of uses (or **attempted** uses) of these tests.

Review

Absolute convergence test

If $\sum_{k}^{\infty} \left| b_k \right|$ converges then $\sum_{k}^{\infty} b_k$ converges.

Root and Ratio tests:

Given any series $\sum_{k}^{\infty} b_k$: **evaluate the limit** of either/or

- the ratio of successive terms (**ratio** test), ignoring any signs:

  $L = \lim_{k \to \infty} \left| \frac{b_{k+1}}{b_k} \right|$

- the k^{th} root, ignoring any signs (**root** test), of the k^{th} term:

  $L = \lim_{k \to \infty} \sqrt[k]{\mid b_k \mid}$.

Then

- If $L < 1$, then $\sum_{k}^{\infty} b_k$ converges.
- If $L > 1$, then $\sum_{k}^{\infty} b_k$ diverges.
- If $L = 1$, then you must use another convergence test. The series doesn't compare to a geometric series.

Exercises to submit

1. Let's start easy: suppose that you're asked to consider the convergence of a geometric series:

$\sum_{k=0}^{\infty} a\, r^k$.

- *a*. Try the root test
- *b*. Try the ratio test
- c. State your conclusion. (What about the cases "on the boundary"?)

- d. Was one test easier than the other?

2. Still easy, but maybe for the wrong reason: suppose that you're asked to consider the convergence of the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$. We know that this is conditionally convergent (by the AST), but not absolutely convergent (since the harmonic series diverges). What do the root and ratio tests tell us?

- *a*. Try the root test (you need to actually calculate the limit).
- *b*. Try the ratio test (you need to actually calculate the limit).
- c. What can you conclude from these tests?
- d. Was one test easier than the other?

3. Still easy, but again for the wrong reason: suppose that you're asked to consider the convergence of the series $\sum_{k=1}^{\infty} \frac{1}{k^2}$. We know that this is absolutely convergent (as a positive p-series). What do the root and ratio tests tell us?

- *a*. Try the root test (you need to actually calculate the limit).
- *b*. Try the ratio test (you need to actually calculate the limit).
- c. What can you conclude from these tests?
- d. Was one test easier than the other?

4. As we will soon see there are series representation for many well-known functions. However, if the series do not converge, the series cannot represent the function.

- *a.* $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$, for any x where the series converges.
  - Determine if this series converges or not if $x = -1$. If it does use a partial sum, along with an appropriate error estimate, to approximate e^{-1} with error less than 0.0001.
  - Determine if this series converges or not if $x = -2$. If it does use a partial sum, along with an appropriate error estimate, to approximate e^{-2} with error less than 0.0001.
- *b.* $\ln(x) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{(x-1)^k}{k}$, for any x where the series converges.
  - Determine if this series converges or not if $x = 0.5$. If it does, use a partial sum, chosen using an appropriate error estimate, to approximate ln(0.5) with error less than 0.0001.
  - Determine if this series converges or not if $x = 2$. If it does, use a partial sum, chosen using an appropriate error estimate, to approximate ln(2) with error less than 0.0001.