Lab 3: Student Assignment

Week 3, January 25-31

MAT 229, Spring 2021

Special Constants

Standard notation	Mathematica notation
$\pi \approx 3.14159$	**Pi**
$e \approx 2.71828$	**E**

Commands

Functionality	Mathematica notation
plot the graph of a function	**Plot[...]**
square root of something, $\sqrt{\ldots}$	**Sqrt[...]**
absolute value of something, $\lvert \ldots \rvert$	**Abs[...]**
sine of something (radian mode), $\sin(\ldots)$	**Sin[...]**
cosine of something (radian mode), $\cos(\ldots)$	**Cos[...]**
tangent of something (radian mode), $\tan(\ldots)$	**Tan[...]**
natural logarithm of something, $\ln(\ldots)$	**Log[...]**
inverse sine of something, $\sin^{-1}(\ldots)$	**ArcSin[...]**
inverse cosine of something, $\cos^{-1}(\ldots)$	**ArcCos[...]**
inverse tangent of something, $\tan^{-1}(\ldots)$	**ArcTan[...]**
integrate something, $\int(\ldots)$	**Integrate[...]**

Exercises to submit

Exercise 1

The linear approximation for a function $f(x)$ at $x = a$ is the $m\,x + b$ that comes from the tangent line $y = m\,x + b$ to $f(x)$ at $x = a$. It provides a simple approximation to $f(x)$ for values of x near a.

$$f(x) \approx m\,x + b$$

Let $f(x) = \tan^{-1}(4\,x) - 2\,x + 3$.

- Define this function in Mathematica.

- Define x0=1/2 (if you use a semi-colon after your definition, nothing will print -- but the parameter x0 will be defined. Sometimes you don’t want to see so much stuff....).
- Determine the linear approximation for $f(x)$ at $x =$ x0.
- Define this linear function as g(x) in Mathematica.
- Plot both f(x) and g(x) on [-1,2].
- What is the absolute value (decimal) of the difference between $f(1)$ and $g(1)$? In other words, how bad is the approximation at x=1?
- What is the absolute value (decimal) of the difference between $f(2)$ and $g(2)$? In other words, how bad is the approximation at x=2?

Exercise 2

Let $f(x) = \sin^{-1}\left(\frac{x}{2}\right) + \ln(x^2 + 1) - 2$.

- Define this function in Mathematica.
- Use your function to determine the y-intercept of $y = f(x)$. Get a decimal value.
- Get all real-valued critical numbers for $f(x)$ as decimal values. Name them xleft and xright, where xleft<xright.
- Evaluate $f(x)$ at your critical numbers to determine the y-values of the critical points.
- Evaluate $f''(x)$ at your critical numbers to determine if each is a local maximum or a local minimum.

  Local minimum points:

  Local maximum points:

- Determine the inflection points for this function, and name it xinf.
- Make a plot of the graph of $f(x)$ to verify your calculations for the y-intercept, the local max/min points, and inflection points. If you’d defined everything correctly, then you’ll see everything looking beautiful below:

In[•]:=
```
Show[
 Plot[f[x], {x, -3, 1}],
 ListPlot[{{xleft, f[xleft]}, {xright, f[xright]}, {xinf, f[xinf]}}]
]
```

Exercise 3

Let $g(x) = \frac{1}{a+x^2}$ and $h(x) = \frac{x^2}{b+x}$. Define these in Mathematica. Then set a to 1 and b to 2.

- Plot the graphs of $g(x)$ and of $h(x)$ together on the same axes for various ranges of x-values until you have one that clearly shows the region bounded by these two graphs. Make sure to zoom in on the action!

- Get decimal numbers for the x-values of the intersection points. Name them xleft and xright.
  Intersection points: $x \approx$

- Modify the "Show" command from Exercise 2, to include the intersection points. You'll know if you've got them right!

- Find the area of the region bounded by $y = g(x)$ and $y = h(x)$.
  Area:

- Does your area make sense? Can you approximate it, and are you close?

Exercise 4

Let $p(x) = \ln\left(\frac{3}{1+x^2}\right)$

- Plot this function to see the region bounded by the x-axis and $y = p(x)$.

- Get the x-intercepts.
  Intersection point: $x \approx$

- Sketch (you could use Mathematica) the solid obtained by rotating the region bounded by $y = p(x)$ and the x-axis about the line y=-1.

- Find the volume of this solid:
  Volume: