Lab 5: Student Assignment

Week 4, February 8

MAT 229, Spring 2021

Exercises to submit

These must be done by hand on paper (or typed into Mathematica). Use Mathematica to check your work. Submit as pdf files in Canvas.

Exercise 1

Evaluate $\int x \sec^2(x)\, dx$. You will want to use more than one technique of integration.

Exercise 2

Evaluate $\int \sin^3(x) \cos^3(x)\, dx$.

a. Use the substitution $u = \sin(x)$.

b. Use the substitution $u = \cos(x)$.

c. Why are these the same answer?

Exercise 3

Consider the indefinite integral $\int \sec(x)\, dx$. In our materials for the week, we asked you to make a sly move to evaluate this integral: first write the integrand $\sec(x)$ as $\sec(x)\, \frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}$.

(Who would think of this? Very mysterious (perhaps space aliens). But the trick works!)

1. Multiply the sec(x) times the numerator of that ratio $\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}$ (which is just a special form of one); notice that the resulting numerator is just the derivative of the denominator, and suggest a suitable u-substitution.
2. Now evaluate the integral. Your answer should have **an absolute value** in it. (If not, figure out where one goes! Ask us for help, if you don't know.)
3. Define the function g(x) as the particular antiderivative of sec(x) you found which passes through the origin (in other words, choose the arbitrary constant C so that your answer satisfies g(0)=0).
4. Once you've defined g(x), execute the following Mathematica command. You should see three functions:

4.1. Mathematica's solution to the antiderivative of sec(x) (in blue),

4.2. your solution (dashed), and

4.3. the original function sec(x) (the integrand, "thin").

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In[]:= f[x_] = Integrate[Sec[x], x]
Plot[{f[x], g[x], Sec[x]}, {x, -Pi, Pi}, PlotStyle → {Blue, Dashed, Thin}]
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5. What’s wrong with Mathematica’s “solution”?

Exercise 4

Let $g(x) = \sin(x)$.

1. What is a range of x-values that give one arch of this sine curve above the x-axis?

2. Rotate the region between the x-axis and this arch about the horizontal line $y = -1$ to get a solid of revolution.

What is the shape of the cross sections?

Set up the integral that gives the volume of this solid.

3. Using an appropriate integration technique, compute this integral by hand to find the volume of this solid.

4. To make sure your answer is reasonable, approximate this solid with a solid cylinder that has had a core cylinder removed; just compute its volume using the volume of cylinder formula $V = \pi r^2 h$.