Lab 8 Notes (eight is just a sideways ∞)

$$\int_0^\infty e^{-3x}\,dx = \lim_{R\to\infty} \underbrace{\int_0^R e^{-3x}\,dx}_{\text{an ordinary definite integral}}$$

$$= \lim_{R\to\infty} \left[-\frac{1}{3} e^{-3x} \right|_0^R$$

$$= \lim_{R\to\infty} \left[-\frac{1}{3} e^{-3R} + \frac{1}{3} e^{0} \right]$$

$$= \underbrace{\lim_{R\to\infty} \left(-\frac{1}{3} e^{-3R} \right)}_{} + \frac{1}{3}$$

$$= 0 + \frac{1}{3} = \boxed{\frac{1}{3}}$$

$$\int_2^{\infty} \frac{x}{x^2-1}\,dx = \lim_{R\to\infty} \underbrace{\int_2^{R} \frac{x}{x^2-1}\,dx}$$

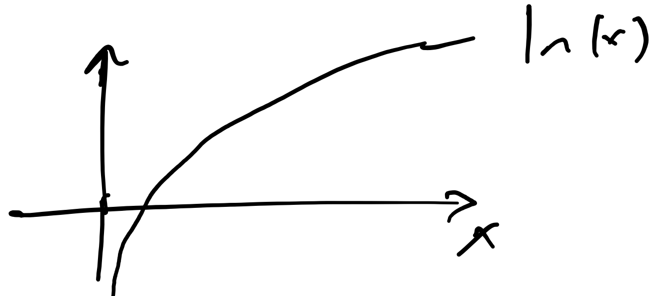
$$= \lim_{R\to\infty} \int_{x=2}^{x=R} \frac{\frac{1}{2}du}{u} \qquad \begin{aligned} u &= x^2-1 \\ du &= 2x\,dx \\ \frac{du}{2} &= x\,dx \end{aligned}$$

$$= \lim_{R\to\infty} \frac{1}{2}\ln|u| \Big|_{x=2}^{x=R}$$

$$= \lim_{R\to\infty} \frac{1}{2}\ln|x^2-1| \Big|_2^R$$

$$= \lim_{R\to\infty} \left[\frac{1}{2}\ln|R^2-1| - \frac{1}{2}\ln 3 \right]$$

$$= \underbrace{\lim_{R\to\infty} \frac{1}{2}\ln|R^2-1|} - \frac{1}{2}\ln 3$$



diverges to ∞

$$\int_0^1 \frac{x}{1-x^2}\,dx = \lim_{r \to 1^-} \int_0^r \frac{x}{1-x^2}\,dx$$

$$= \lim_{r \to 1^-} \left[-\int_0^r \frac{x}{x^2-1}\,dx \right]$$ looks familiar!

$$= \lim_{r \to 1^-} \left[-\left(\frac{1}{2} \ln|x^2-1| \right) \Big|_0^r \right]$$

$$= \lim_{r \to 1^-} \left[-\frac{1}{2} \ln|r^2-1| + \cancel{\frac{1}{2} \ln(1)}^{\,0} \right]$$

$$= \underbrace{\lim_{r \to 1^-} \left(-\frac{1}{2} \ln|r^2-1| \right)}_{\text{blowing up to } \infty}$$

divergent

$$\int_0^1 \frac{x+1}{\sqrt{x^2+2x}}\,dx = \lim_{r\to 0^+}\int_r^1 \frac{x+1}{\sqrt{x^2+2x}}\,dx$$

$$= \lim_{r\to 0^+}\int_{x=r}^{x=1} \underbrace{\frac{1}{2}u^{-1/2}}\,du$$

$u = x^2+2x$

$du = (2x+2)\,dx$

$= 2(x+1)\,dx$

$\frac{du}{2} = (x+1)\,dx$

$$= \lim_{r\to 0^+}\left[\frac{u^{1/2}}{\ } \right]_{x=r}^{x=1}$$

$$= \lim_{r\to 0^+}\left[(x^2+2x)^{1/2} \right]_r^1$$

$$= \lim_{r\to 0^+}\left[3^{1/2} - (r^2+2r)^{1/2} \right]$$

$$= 3^{1/2} - \underbrace{\lim_{r\to 0^+}(r^2+2r)^{1/2}}$$

$$= 3^{1/2}$$

convergent