Lab 9

MAT 229, Spring 2021

Exercises to submit

These must be done by hand on paper (or typed into Mathematica).
Use Mathematica to check your work. Submit as pdf files in Canvas (if you can make it a single pdf, that helps the graders immensely).

Exercise 1

Let $f(x) = \sin(x)$. We want to approximate $f(x)$ with Taylor polynomials centered at 0 for values of x in $[-\pi/4, \pi/4]$.

a. Find the degree n such that you know the error in approximating $f(x)$ with $T_n(x)$ is less than 0.00001.

b. What is $T_n(x)$?

c. What is the Taylor series error estimate for x in $[-\pi/4, \pi/4]$?

Exercise 2

Let $g(x) = \ln(x)$. We want to approximate $g(x)$ with Taylor polynomials centered at 1.

a. Find the degree n such that you know the error in approximating $\ln(1.3)$ with $T_n(1.3)$ is less than 0.0001.

b. Repeat the same question with a different input value. Find the degree n such that you know the error in approximating $\ln(1.4)$ with $T_n(1.4)$ is less than 0.0001.

c. The Taylor's error estimate says that when approximating $f(x)$ with $T_n(x)$ for $|x - a| \le d$ (in other words $-d \le x - a \le d$) choose K in

$$|R_n(x)| \le \frac{K}{(n+1)!} |x - a|$$

so that $K \ge \left| f^{(n+1)}(x) \right|$ whenever $|x - a| \le d$. For $g(x) = \ln(x)$ and $a = 1$, how big must d be for it to be impossible to find K?

Exercise 3

The fundamental theorem of calculus has two parts. One part is that if

$$F(x) = \int_a^x f(t)\, dt$$

then

$$F'(x) = f(x).$$

Let $F(x) = \int_0^x e^{-t^2}\, dt$. Approximate it with Taylor polynomials centered at 0.

a. What is $F(0)$? (This is easy; look at the integral.)

b. What is $T_4(x)$?

c. What is an error estimate in using this Taylor polynomial to approximate $\int_0^1 e^{-t^2}\, dt$.

d. Use the midpoint rule with $n = 4$ to

d.a. Approximate $F(1) = \int_0^1 e^{-t^2}\, dt$.

d.b. Compare to the value $T_4(1)$ and the actual value $F(1)$.

d.c. According to the midpoint rule error estimate, what is an error estimate for this approximation?

d.d. Compare to the actual absolute error of the midpoint estimate.

e. Here's a surprise for you:

e.a. What is $T_3(x)$?

e.b. Compare to $T_4(x)$ in part a., and explain (symmetry!).

e.c. Without doing any calculations, how are $T_{400}(x)$ and $T_{399}(x)$ related?