Weekly Assignment 1

1. Average value

The average of a discrete number of values x_1, x_2, ..., x_n is $\frac{1}{n}\sum_{k=1}^{n} x_k$. The average of a continuous range of values, $f(x)$, $a \le x \le b$, is $\frac{1}{b-a}\int_a^b f(x)\,dx$. Use this second fact to work the following.

Find the average value of

- $f(x) = e^{x/2}$, $0 \le x \le 2$

- Graph the above function and use it to estimate the x-value for which $f(x) =$ its average value.
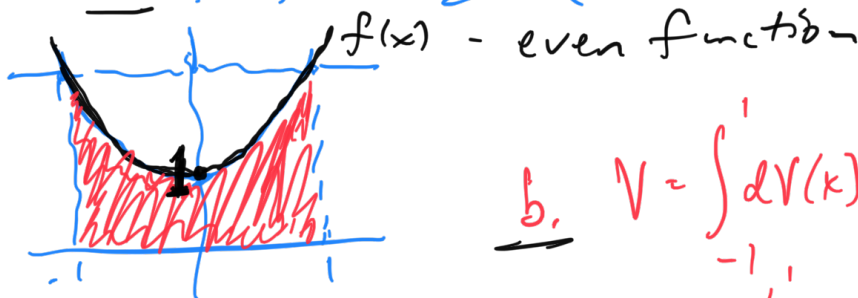
2. Volume

Find the volume of the solid of revolution obtained by rotating the region bounded by $y = \frac{e^x + e^{-x}}{2}$ and the x-axis for $-1 \le x \le 1$, rotated about the x-axis.

a. Sketch the planar region before rotating.

b. Set up the integral(s) you use for this volume.

c. Evaluate the integral(s) and find the volume.

#2 a. $f(x) = \frac{e^x + e^{-x}}{2}$ $\left(= \cosh(x)\right)$

"hyperbolic cosine" – cool function! More later....

$f(x)$ – even function

b. $V = \int dV(x) = \int_{-1}^{1} A(x)\,dx$

$= \int_{-1}^{1} \pi \left(\frac{e^x + e^{-x}}{2}\right)^2 dx$

c. $V = \frac{\pi}{4}\int_{-1}^{1}\left[(e^x)^2 + 2e^x e^{-x} + (e^{-x})^2\right]dx = \frac{\pi}{4}\int_{-1}^{1}\left(e^{2x} + e^{-2x} + 2\right)dx$

$= \frac{\pi}{4}\left[\frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} + 2x\right]_{-1}^{1} = \frac{\pi}{4}\left[\frac{1}{2}(e^2 - e^{-2}) + 2 - \left(\frac{1}{2}(e^{-2} - e^{2}) - 2\right)\right]$

$= \frac{\pi}{4}\left[4 + e^2 - e^{-2}\right] \approx 8.8365$

$\left(\begin{array}{l}\text{Does that} \\ \text{look close?} \\ \text{I say} \\ \text{yes!}\end{array}\right)$