Section Summary: 7.3: Trigonometric Substitution

a. **Definitions** None to speak of.

b. **Theorems**

c. Properties/Tricks/Hints/Etc.

By making trig substitutions, we are essentially using the u-substitution in reverse (another chance to do something backwards!).

In this case, we replace the dummy variable of integration (usually x) with some function (in this case trig functions). The purpose in each case is to eliminate a pesky square root....

So if you're looking at terms of the form

$$\sqrt{a^2 - x^2}$$

then try $x = a\sin(\theta)$.

If you're looking at terms of the form

$$\sqrt{a^2 + x^2}$$

then try $x = a\tan(\theta)$.

And if you're looking at terms of the form

$$\sqrt{x^2 - a^2}$$

then try $x = a\sec(\theta)$.

In every event, these substitutions may make the integral you encounter upon substitution a little nicer than the original.

In the end, once you've solved the integral in terms of θ, we need to rethink the trig functions in terms of the triangle (see the figures in this section). This is so that we can rewrite the indefinite integral in terms of x (rather than θ). So be prepared to use some right triangle of your own creation.

d. Summary

We use trig functions, in particular certain identities among them, to rewrite integrals in forms more amenable to solution.

Once again we find new ways to solve integrals analytically.