MAT229 Test 1 (Spring 2019): New Functions Over Calc I

Name:

Directions: Weights for problems are not equal. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it).

Don't erase (cross out, instead); that way, in case you've done something good, I can give you credit. I have scratch paper up front.

Good luck!

Problem 1. Consider $f(x) = 2^{(3x-1)^2}$.

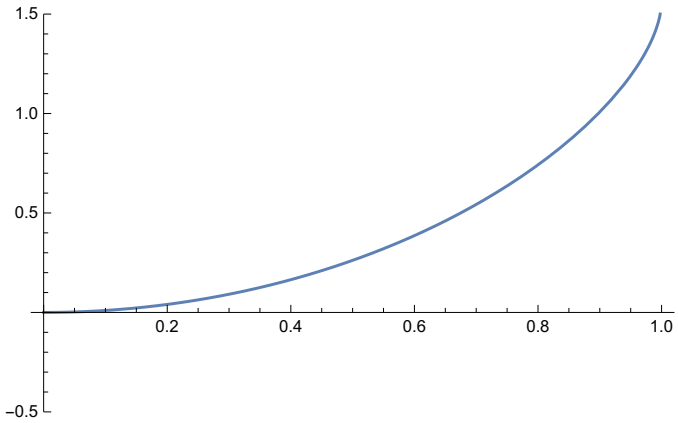
a. (6 pts) Solve the equation $f(x) = 16$ for x.

b. (4 pts) Is the function f invertible? Why or why not?

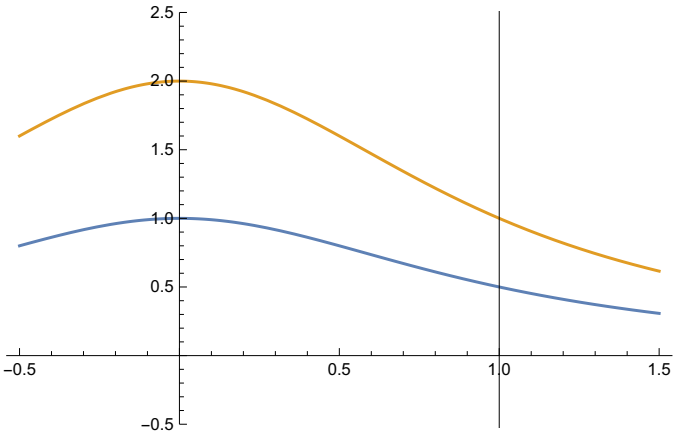
Problem 2. Let $f(x) = x\sin^{-1}(x)$, and let me remind you that $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.

a. (8 pts) Find an equation of the tangent line to the graph of f when $x = \frac{1}{2}$.

b. (4 pts) Carefully graph your tangent line into the plot below:



Problem 3. (10 pts) Find the area of the region bounded by the graphs of $y = \dfrac{1}{x^2+1}$ and $y = \dfrac{2}{x^2+1}$, and the lines $x = 1$ and the y-axis.



Problem 4. Let $f(x) = \ln\left(\tan^{-1}(x-1)\right)$.

a. (6 pts) What is the domain of $f(x)$?

b. (6 pts) What is the range of $f(x)$?

c. (6 pts) Find a formula for the inverse function $f^{-1}(x)$.

Problem 5. Let $g(x) = x^2 - \ln\left(x^2\right)$.

a. (8 pts) Find exact values for any critical numbers to $g(x)$. Determine whether each is an absolute or local extremum (max or min), or not. Give reasons for your conclusions.

b. (6 pts) Find intervals of concavity for $g(x)$. Give reasons for your conclusions.

Problem 6. (10 pts) Use the limit definition of the derivative to compute the derivative $f'(x)$, where

$$f(x) = e^{2x}.$$

You may assume that $f'(0) = 2$.

Problem 7. (4 points each) Find the limit, if it exists. Show your work.

a. $\lim_{x \to 0} \dfrac{x^2}{\cos(x)}$

b. $\lim_{x \to -1} \dfrac{x+1}{\sin(x+1)}$

c. $\lim_{x \to \infty} x \tan^{-1}\left(\dfrac{2}{x}\right)$

d. $\lim_{x \to \infty} \left(1 + \dfrac{r}{x}\right)^x$

Problem 8. (10 pts) Find the volume of the solid obtained by rotating about the x-axis the region bounded by $y = \dfrac{1}{\sqrt{x^2+1}}$ and the x-axis for $0 \leq x \leq 1$.