Weekly Assignment #7

MAT 229, Spring 2021

Instructions: **Show your work!**

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1. Recursively defined sequence

a. Determine whether the *sequence* defined as follows is convergent or divergent.

$a_1 = 1$

$a_n = 4 - a_{n-1}$ for $n > 1$

If it converges, what does it converge to. If its diverges, describe how it diverges. For example, does it converge to ∞ or $-\infty$? Does it stay bounded? Does it oscillate?

In[∘]:=
```
a[k_] := If[IntegerQ[k], If[OddQ[k], 1, 3]]
Table[a[k], {k, 1, 10}]
```

Out[∘]=
```
{1, 3, 1, 3, 1, 3, 1, 3, 1, 3}
```

This one oscillates between two values, so it diverges, but stays bounded.

b. Answer the same questions for this same recursive definition $a_n = 4 - a_{n-1}$ but with first term $a_1 = 2$

In[∘]:=
```
a[k_] := If[IntegerQ[k], If[OddQ[k], 2, 2]]
Table[a[k], {k, 1, 10}]
```

Out[∘]=
```
{2, 2, 2, 2, 2, 2, 2, 2, 2, 2}
```

This one is a constant sequence, so it converges to 2, and is bounded.

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2. Geometric series

a. Find the value of b such that $\sum_{k=1}^{\infty} \left(\frac{1}{1+b}\right)^k = 2$.

The formula for a geometric series, $\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$, works when we start indexing from 0 -- so we have to play around a little. $\sum_{k=1}^{\infty} \left(\frac{1}{1+b}\right)^k = \sum_{k=0}^{\infty} \left(\frac{1}{1+b}\right)^k - 1 = 2$, so we have to solve $\sum_{k=0}^{\infty} r^k = 3 = \frac{1}{1-r}$, and then solve for b. In this case, $r = \frac{1}{1+b} = \frac{2}{3}$; then $b = \frac{1}{2}$.

In[∘]:=
```
b = 1/2;
Sum[(1/(1 + b))^k, {k, 1, Infinity}]
```

Out[∘]=
```
2
```

2. Express the repeating decimal number 0.467467467 .... as a ratio of integers by first writing it as a geometric sum.

0.467467467 = 467*.001+467*.000001+467*.000000001+.... = $\frac{467}{1000}\left(1+\frac{1}{1000}+\left(\frac{1}{1000}\right)^2+....\right) = \frac{467}{1000} * \frac{1000}{999} = \frac{467}{999}$

In[∘]:= $N\left[\frac{467}{999}\right]$

Out[∘]= 0.467467

3. Repeat the second part with the repeating decimal number 0.999 .... What’s the surprise?

0.999 = 9*.1+9*.01+9*.001+.... = $\frac{9}{10}\left(1+\frac{1}{10}+\left(\frac{1}{10}\right)^2+....\right) = \frac{9}{10} * \frac{10}{9} = 1$

That’s Crazy! That’s INSANE!

3. Partial sums

1. If the n^{th} partial sum of series $\sum_{k=1}^{\infty} a_k = a_1 + a_2 + a_3 + \ldots$ is $S_n = 3 - \frac{n}{2^n}$. Does the series converge? If so, to what value?

Yes, the series converges, since Limit as n -> Infinity of $S_n = 3 - \frac{n}{2^n}$ **is equal to 3. So the series is equal to 3, as the limit of the partial sums.**

2. Write the first four partial sums of $\sum_{k=1}^{\infty}\left(\frac{k}{2^k} - \frac{k+1}{2^{k+1}}\right)$. What is the form of S_n? Does the series converge? If so, to what value?

$S_1 = \sum_{k=1}^{1}\left(\frac{k}{2^k} - \frac{k+1}{2^{k+1}}\right) = \frac{1}{2^1} - \frac{2}{2^2}$

$S_2 = \sum_{k=1}^{2}\left(\frac{k}{2^k} - \frac{k+1}{2^{k+1}}\right) = \left(\frac{1}{2^1} - \frac{2}{2^2}\right) + \left(\frac{2}{2^2} - \frac{2+1}{2^{2+1}}\right) = \frac{1}{2} - \frac{3}{2^3}$

$S_3 = \sum_{k=1}^{3}\left(\frac{k}{2^k} - \frac{k+1}{2^{k+1}}\right) = \left(\frac{1}{2} - \frac{3}{2^3}\right) + \left(\frac{3}{2^3} - \frac{4}{2^4}\right) = \frac{1}{2} - \frac{4}{2^4}$

$S_4 = \sum_{k=1}^{4}\left(\frac{k}{2^k} - \frac{k+1}{2^{k+1}}\right) = \frac{1}{2} - \frac{5}{2^5}$

$S_n = \sum_{k=1}^{n}\left(\frac{k}{2^k} - \frac{k+1}{2^{k+1}}\right) = \frac{1}{2} - \frac{n+1}{2^{n+1}}$

It must converge to $\frac{1}{2}$**, since** $\frac{n+1}{2^{n+1}}$ **converges to 0, as n-> Infinity.**