

Weekly Assignment #7

MAT 229, Spring 2021

Instructions: **Show your work!**

1. Recursively defined sequence

- a. Determine whether the *sequence* defined as follows is convergent or divergent.

$$a_1 = 1$$

$$a_n = 4 - a_{n-1} \text{ for } n > 1$$

If it converges, what does it converge to. If it diverges, describe how it diverges. For example, does it converge to ∞ or $-\infty$? Does it stay bounded? Does it oscillate?

- b. Answer the same questions for this same recursive definition $a_n = 4 - a_{n-1}$ but with first term $a_1 = 2$
-

2. Geometric series

- a. Find the value of b such that $\sum_{k=1}^{\infty} \left(\frac{1}{1+b}\right)^k = 2$.

- b. Express the repeating decimal number $0.467467467 \dots$ as a ratio of integers by first writing it as a geometric sum.

- c. Repeat the second part with the repeating decimal number $0.999 \dots$. What's the surprise?
-

3. Partial sums

- a. If the n^{th} partial sum of series $\sum_{k=1}^{\infty} a_k = a_1 + a_2 + a_3 + \dots$ is $S_n = 3 - \frac{n}{2^n}$. Does the series converge? If so, to what value?

- b. Write the first four partial sums of $\sum_{k=1}^{\infty} \left(\frac{k}{2^k} - \frac{k+1}{2^{k+1}}\right)$. What is the form of S_n ? Does the series converge? If so, to what value?