Integration By Parts Worksheet

Integration by parts

Let’s say you **don’t** like the integral $\int f(x)\, g'(x)\, dx$. Integration by parts allows you to rewrite it as

$$f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

if you like, and maybe that new integral on the right will look better to you (replace one integral for another). This is the product rule for differentiation, in reverse!

Note that this works for definite integrals, too: we simply add limits:

$$\int_a^b f(x)\, g'(x)\, dx = f(x)\, g(x) \Big|_a^b - \int_a^b f'(x)\, g(x)\, dx$$

So you've got choices. The first thing to look for in the integrand is a **product,** of two functions:

one you wouldn't mind differentiating (*f(x)*), and

the other you wouldn't mind **anti**-differentiating (think of it as *g'(x)*).

Alternate form

If we write *u=f(x)* and *dv=g'(x)dx*, then *du=f'(x)dx* and *v=g(x);* and if we can identify an integral as $\int u\, dv$, then we can rewrite the integration by parts formula as

$$\int u\, dv = u\, v - \int v\, du.$$

This makes it all look a little like a double substitution. It’s actually just a good shorthand. I personally prefer the first form we considered, above -- but you’re welcome to use this alternative form (and sometimes I do!).

Problems to work on:

1. To evaluate $\int x^n \ln(x)\,dx$, use integration by parts with $f(x) = \ln(x)$ and $g'(x) = x^n$.
2. Using your results from problem one, what is $\int \ln(x)\,dx$?
3. Evaluate $\int_0^\pi x^2 e^{-4x}\,dx$ using integration by parts.
4. Evaluate $\int x^3 \cos(x^2)\,dx$ using integration by parts, but first make a substitution.
5. What is the area of the region bounded by $y = \sin^{-1}(x)$, the x-axis, and $x = 1/2$? (How did we find the anti-derivative of $\tan^{-1}(x)$?)
6. What is the volume of the solid obtained by rotating about the x-axis the region bounded by $y = x\sqrt{\ln(x)}$ and the x-axis for $1 \le x \le e$. (Use problem one!)
7. In problem one above, there is one special case $n = -1$. Use integration by parts in this particular instance to get $\int x^{-1} \ln(x)\,dx = \text{stuff} - \int x^{-1} \ln(x)\,dx$. Solve this equation for $\int x^{-1} \ln(x)\,dx$ to finish evaluating the integral. (Can you think of how this becomes a general rule? What is special about the integrand, $x^{-1} \ln(x)$?)
8. To evaluate $\int e^x \cos(x)\,dx$, use integration by parts **twice**. (Be sure to choose u and dv the same way both times. If you choose $u = e^x$ the first time, be sure to choose $u = e^x$ the second time. Or, if you choose u = trig function the first time, choose u = trig function the second time.) Then employ what you did in problem 7 to finish evaluating the integral.