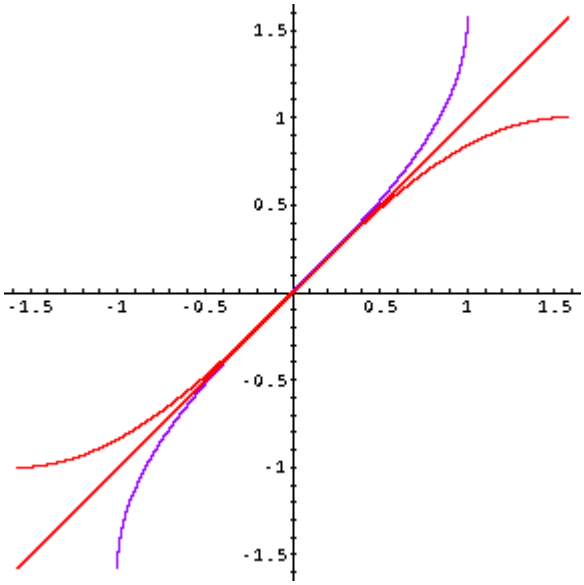
Inverse trigonometric functions

a. Definitions

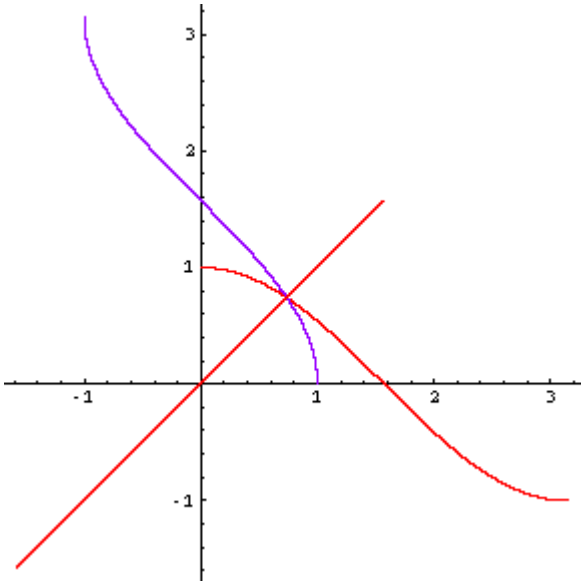
- **arcsine** or $\sin^{-1}(x)$: the inverse function of sine (usually taken on the interval $[\frac{-\pi}{2}, \frac{\pi}{2}]$ by default).



$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

This function (and the other inverse trig functions) could be defined on intervals other than those given above. There are an infinite number of "inverses of sine", for example, depending on which interval we choose (sine simply needs to be one-to-one on that interval).

- **arccosine** or $\cos^{-1}(x)$: the inverse function of cosine (usually taken on the interval $[0, \pi]$ by default).



$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

Suppose that we want to find the derivative of $\cos^{-1}$, which we'll do using implicit differentiation:

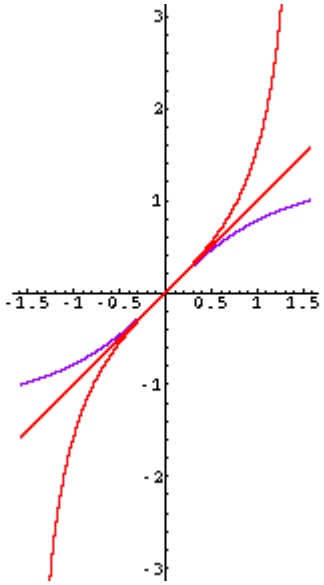
$$\cos^{-1}(x) = y \Longrightarrow x = \cos(y)$$

$$\frac{d}{dx}\cos(y) = -\frac{dy}{dx}\sin(y) = 1$$

$$\frac{dy}{dx} = \frac{-1}{\sin(y)} = \frac{-1}{\sqrt{1-\cos^2(y)}} = \frac{-1}{\sqrt{1-x^2}}$$

Note that we chose the positive sign on the square root, because the derivative of arccos is clearly negative (since arccos is decreasing). The same method of implicit differentiation is demonstrated in the text, to find the derivatives of arcsine and arctan.

- **arctangent** or $\tan^{-1}(x)$: the inverse function of tangent (usually taken on the interval $[\frac{-\pi}{2}, \frac{\pi}{2}]$ by default).



$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

b. Summary

The trigonometric functions are obviously very important functions, and their "inverses" (obtained by domain restriction) are likewise useful. (They are not, however, in the same class as the exponential and logarithmic functions; at least not in my experience. And I've never had need for the other arc functions – e.g. of secant).

All these functions do appear as antiderivatives of certain classes of functions, as you can see from the derivatives of these functions. Spend a little time getting to know them.

Look at the graphs of the functions $\sin^{-1}(x)$ and $\cos^{-1}(x)$: can you write one of these functions in terms of the other, using only reflections, shifts, etc.?