3D Coordinates

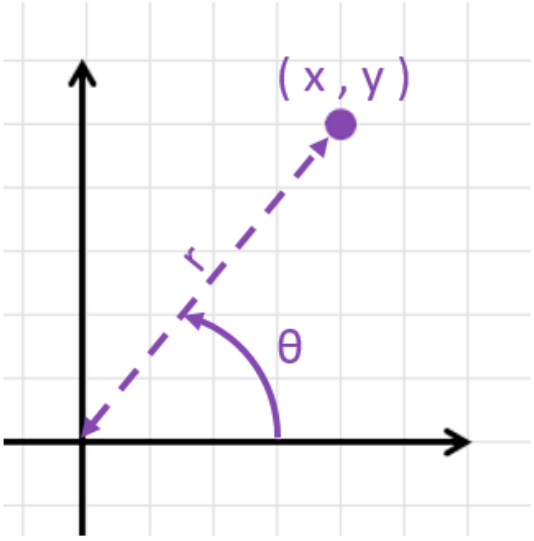
MAT 229, Spring 2021

Week 15

- Stewart's *Calculus*
  Section 12.1: Three-Dimensional Coordinate Systems
- Calculus, Volume 3 (Authors: Gilbert Strang and Edwin "Jed" Herman)
  Chapter 9. Vectors in Three Dimensions

2D Coordinates

For points in the plane we have Cartesian coordinates (x, y) and polar coordinates (r, θ). Two numbers are needed to address any point.



$$\begin{cases} x = r.\cos(\theta) \\ y = r.\sin(\theta) \end{cases}$$

$$\begin{cases} r = \sqrt{x^2 + y^2} \\ \theta = \operatorname{atan2}(y, x) \end{cases}$$

(source)

Question

How are locations on Earth's surface typically represented? (Video)

3D Coordinates

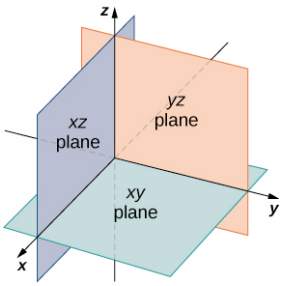
Question

What information is needed to locate the position of a flying plane?

Cartesian coordinates in 3D

Start with the x-y plane. Add depth with the z-axis coming out perpendicularly from the plane (that is, at an angle of 90°). A point in space has coordinates (x, y, z) where

- z is the distance of the point from the x-y plane
- y is the distance of the point from the x-z plane
- x is the distance of the point from the y-z plane



Questions

- The equation $z = 3$ is the set of points (x, y, z) with $z = 3$. What is the shape of this set? (Video)
- The equation $x = 2$ is the set of points (x, y, z) with $x = 2$. What is the shape of this set? (Video)
- Give an equation for the plane that is parallel to the y-z plane and is 5 units from it in the positive x direction.
- Sketch the equation $x + y = 3$. (Video)

  What does the fact that the equation is independent of z tell you?
- The equation $y > 1$ is the set of points (x, y, z) with $y > 1$. What is the shape of this set? (Video)

  What does the fact that this equation is independent of **both** x and z tell you?

Distance

Questions

We want to find the distance between (1, 2, 0) and (2, 1, 3).

- Draw a box with one corner at (1, 2, 0) and the diagonal corner at (2, 1, 3).

- What are the dimensions of this box?
- What is the distance between the two points?

(Video)

3D distance

The distance between points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}\ .$$

Notice how this compares to the distance formula for points in the plane.

Questions

Consider the triangle whose vertices are $(3, -2, -3)$, $(7, 0, 1)$, and $(1, 2, 1)$.

- Is it a right triangle?
- Is it an isosceles triangle?

(Video)

Questions

Consider the set of points (x, y, z) that are a distance of 2 from the origin $(0, 0, 0)$.

- What is an equation that x, y, z must satisfy for (x, y, z) to be in this set?
- What is this shape?

(Video)

Spheres

From the distance formula, we can deduce an equation of the sphere centered on *C(a,b,c)* of radius *r*. The sphere is the set of all points **equidistant**, at a distance *r*, from the center.

That is, a sphere is the set of points that are the same distance, the radius, from a specified point -- the sphere's center. If the radius is *r* and the center has coordinates (a, b, c), then this is **an equation for the sphere**:

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2.$$

Questions

- The equation $(x - 2)^2 + y^2 + (z + 3)^2 = 4$ represents a particular sphere.
  - What is its center?
  - What is its radius?
  - Describe its intersections with each of the coordinate planes.

(Video)

- The equation $x^2 + y^2 + z^2 + 2x - 4y - 10z = 0$ represents a sphere.
  - What is its center?
  - What is its radius?

(Video)

Homework

- IMath problems on the Three-Dimensional Coordinate Systems