

Instructions. Write your answers in the space provided (if you need more paper, ask!). Show all your work. Avoid erasing – cross out instead. Who knows – you might have been right!

1. (20 points) Use Gaussian elimination to solve each of the following two linear systems. Express your solution sets in vector form.

$$\begin{aligned} \text{(a)} \quad & 2x - y + z - 2w = 9 \\ & x + 2z - 2w = 8 \\ & 3y - z = 5 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & 4x - y = 10 \\ & x + 2y = 7 \\ & -x + 3y = 3 \end{aligned}$$

2. (20 points) Give an example of each of the following or explain in one or two sentences why an example is impossible:

a. A linear system of three equations with three variables that has all nonzero coefficients and no solutions.

b. A vector \vec{w} perpendicular to $\vec{v} = \begin{pmatrix} -4 \\ 2 \\ 2 \end{pmatrix}$.

c. A linear system of two equations with infinitely many solutions.

d. A homogeneous linear system with no solutions.

3. (20 points) For all parts of this problem, use these:

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 3 \\ 4 & 5 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 \\ -2 & -4 \\ 2 & 5 \end{pmatrix} \quad \vec{u} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix} \quad \vec{v} = \begin{pmatrix} -4 \\ 2 \\ 2 \end{pmatrix}$$

- Compute $\vec{u} \cdot \vec{v}$.
- Find the lengths of \vec{u} and \vec{v} , and the angle between them.
- Is A row equivalent to B ?
- Is the point $(-3, 6, 2)$ on the line determined by $\vec{u} + t\vec{v}$?

4. (20 points) Use Gauss-Jordan reduction to solve the following system of linear equations.

$$a + b - c + d = 4$$

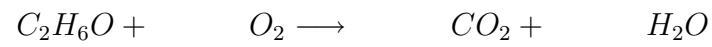
$$b + c + d = 2$$

$$a + 3b + c + 4d = 13$$

$$2a + 5b + c + 6d = 19$$

- a. Reduce to the unique row-equivalent Gauss-Jordan form.
- b. Read the solution off of this form, expressing your solution set in vector form, and identifying the particular solution and solution set of the homogeneous system. What kind of hyperspace is this solution set? What is its dimension?

5. (20 points) Ethanol burns with oxygen to produce carbon dioxide and water. The following equation is unbalanced:



Create a linear system to balance this chemical equation, and find the solution with the smallest integer coefficients.