History of Mathematics

Exercises on infinity.

1. Resolve the Achilles paradox of Zeno. (Achilles, in order to pass the moving tortiose must first cover half the distance between them. Therefore, Achilles can never pass the tortiose.)

2. Explain why the "paradoxes" as set forth by Euler are no longer paradoxes today.

3. The modern definition of the real number is this: The real numbers are defined to be the set of equivalence classes of pairs of rational sequences (a_i, b_i), where (1) $\{a_i\}$ is increasing, (2) $\{b_i\}$ is decreasing, (3) for each $i = 1, 2, \ldots,$ $b_i - a_i > 0$, and (4) $\lim_{i\to\infty}(b_i - a_i) = 0$.
   a. Prove the distributive law of multiplication for numbers.
   b. Describe the representation of π in this sense.

4. Resolve this paradox. Let N be the set of all natural numbers that can be described with a sentence of one hundred characters or less. Clearly there are at most 100^{127} such numbers, where all 127 printable ASCII characters have been permitted.

   Let n be the first number not in the set N.

   This sentence, which describes the number n, has less than one hundred characters. Therefore $n \in N$.

5. Show that the set of continuous functions on an interval has cardinality $\aleph_1$.
6. What is the cardinality of all the one-to-one functions on the interval $[0, 1]$? What is the cardinality of all the monotonic functions on the interval $[0, 1]$?
7. Find a set S for which the third derived set, ala Cantor, $S''' \neq \emptyset$, and not infinite.
8. Give a careful analysis of ancient views of infinity, particularly Aristotle.
9. The paradoxes of infinity have caused many difficulties in the early days of set theory. What are they and how were they resolved?
10. Who is Kurt Godel? What was his impact on infinity and set theory and mathematics in general? How did his work impact Hilbert's program?