8, section 4.4

Note Title 3/3/2006

Here's # 8 in all its glory, without prior simplification to the smallest set of consistent equations.

1. $x \equiv 1 \pmod 2$
2. $x \equiv 2 \pmod 3$
3. $x \equiv 3 \pmod 4$
4. $x \equiv 4 \pmod 5$
5. $x \equiv 5 \pmod 6$
6. $x \equiv 0 \pmod 7$

The first equation yields

$$x = 2k+1 \equiv 2 \pmod 3$$

$$\therefore\ 2k \equiv 1 \pmod 3 \equiv 4$$

$$k \equiv 2 \pmod 3$$

$$x = 2(2+3l)+1$$

$$= 5+6l \equiv 3 \pmod 4$$

Note! equation 5 is covered here, so we would know not to pursue it further.

$$\therefore\ 6l \equiv -2 \pmod 4, \text{ or}$$

$$2l \equiv -2 \pmod 4, \text{ so}$$

$$l \equiv -1 \pmod 2 \equiv 1 \pmod 2$$

Here's the mistake you'll make if you're not careful, which will "lose" some solutions.

$$x = 5+6(1+2m)$$

$$= 11+12m$$

$$x = 11 + 12m \equiv 4 \pmod 5$$

$$\therefore\ 12m \equiv 2m \equiv 8 \pmod 5$$

$$m \equiv 4 \pmod 5$$

$$x = 11 + 12(4 + 5n)$$

$$= 59 + 60n \equiv 5 \pmod 6$$

unnecessary if we picked up on the earlier hint....

$\therefore\ 60n \equiv 6 \pmod n$, or $0 \equiv 0 \pmod n$; i.e., there's no additional constraint imposed by this equation.

$$x = 59 + 60n \equiv 0 \pmod 7$$

$$\therefore\ 60n \equiv 4 \pmod 7$$

$$4n \equiv 4 \pmod 7 \implies n \equiv 1 \pmod 7$$

$$\therefore\ x = 59 + 60(1 + 7s)$$

$$= 119 + 420s, \text{ or}$$

$$\boxed{x \equiv 119 \pmod{420}}$$

119 eggs.

So we could have cut down on our effort a little, by making a few observations, but we ended up getting the right answer. We need to be cautious when cancelling common factors or we might miss some solutions.

If we keep our eyes open, we can see that some equations aren't necessary (e.g. eq 5, which imposed no additional constraint).