MAT310 Test 2, Spring 2006: Chapters 4 and 5

Name:

Directions:

- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).
- Indicate clearly your answer to each problem (e.g., put a box around it).
- I've gone with fewer problems this time: that means that I expect you to do better work on each one. Justify each step: cite theorems, or prove what you need. Don't assume that I'll give you the benefit of the doubt!
- **Good luck!**

Problem 1 (10 pts) Use principles of number theory (rather than guess and check!) to find the missing digit:

$$2x988x44656 = (6 * 247x4)^2$$

Problem 2 (10 pts).

1. Prove that an integer $n > 1$ is prime $\iff$ $(n-2)! \equiv 1 (\text{mod } n)$.
2. If n is a composite integer, show that $(n-1)! \equiv 0 (\text{mod } n)$, except when $n = 4$.

Problem 3 (10 pts). Demonstrate that if $a_1, a_2, \ldots, a_n$ is a complete set of residues modulo n and $\gcd(c, n)=1$, then $ca_1, ca_2, \ldots, ca_n$ is a complete set of residues modulo n. [Hint: along the way you'll probably need to show that distinct numbers ca_i and ca_j are mutually incongruent mod n.]

Problem 4 (10 pts). You and your friends Joey and Billy and Vicky and Sally are trying to divy up your treasure: a huge pile of Lottery tickets for the big $300M drawing. You try cutting the pile five ways, but end up with four tickets left over. So you vote Sally off the island and try again. You cut four ways and have three tickets left over, so you vote Joey off the island.

You continue in this fashion, and discover each time that when you divy up your trove in n ways you have $n-1$ tickets left over. Each time you vote someone off until you are the only one left, and have all those tickets to yourself, heh heh! Then you learn that they've already drawn the winner, that there was only one winning ticket, and that the winning ticket has been turned in. You are friendless, all for nought. You pledge never to play the lottery again.

1. What's the smallest possible number of tickets in the pile, and
2. what are all the possible numbers of tickets in the pile?

Problem 5 (10 pts). The three most recent appearances of Halley's comet were in the years 1835, 1910, and 1986; the next occurrence will be in 2061 (on the centennial of my birth!). Prove that

$$1835^{1986} + 1910^{2061} \equiv 0 (\text{mod } 7)$$

and that

$$1910^{1835} + 2061^{1986} \equiv 0 (\text{mod } 7)$$

Note: Mark Twain was born in 1835, and claimed that he came in on the comet. He also said that he'd go out with it. He said, and I quote: 'The Almighty has said, no doubt: "Now here are these two unaccountable freaks; they came in together, they must go out together."' Mark Twain died in 1910.