MAT320 Test 1: Chapter 15, Sections 1-7

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (35 pts) Consider the function

$$f(x,y) = x^2 e^{xy}$$

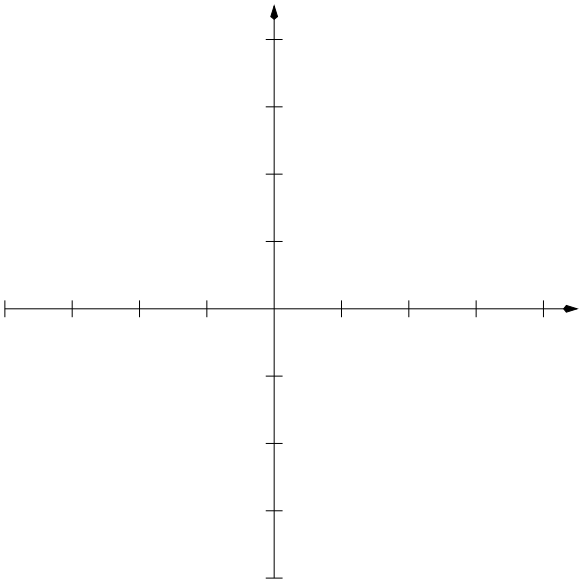
1. (7 pts) Given that
$$\lim_{u \to 0} \frac{e^u - 1}{u} = 1,$$
use the definition of the partial derivative as a limit to calculate f_y.

2. (7 pts) Compute f_x, and find any critical points. What do you notice?

3. (7 pts) Suppose that $x = st$ and $y = ln(st)$. Use the chain rule and a tree (or directed graph) to express $\frac{\partial f}{\partial t}$.

4. (7 pts) Demonstrate that f satisfies Clairaut's theorem.

5. (7 pts) Write an explicit equation $y = l(x)$ for level curves $f(x, y) = c$ in the xy-plane. Draw and label level curves for $c = 0, 1, e$ on the domain $[-2, 2] \times [-2, 2]$. Indicate the orientation



of the gradient along these curves as well.

Problem 2 (15 pts). Consider the function

$$g(x, y) = \frac{x^2 + y^2}{x + y}$$

Discuss the behavior of this function at the origin. In particular, would it be continuous if we set $g(0, 0) = 0$?

Problem 3 (50 pts). Some terrain is modelled by the function

$$g(x, y) = xe^{\frac{-(x^2+y^2)}{2}}$$

1. (25 pts) Find and classify the critical points of g. Be clever!

2. (5 pts) Compute the gradient vector at the point $P(1, 1)$.

3. (10 pts) Standing at the point $P(1,1)$, you consider climbing to the highest point in the landscape. In what direction do you go, and what is the directional derivative in that direction?

4. (10 pts) Find the best linear approximation to f at the point $P(1,1)$. If you were to take the tangent plane, rather than follow the surface of the terrain, what would be the error at the location of the summit?