MAT320 Test 2: Integration, and Lagrange Multipliers

Name:

Directions:

- There are nine problems, equally weighted, that come (pretty much) straight off the homework. I changed a few minor things, and I changed one to make it easier, if you're clever....
- Attempt each problem. Don't leave any blank.
- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).
- Indicate clearly your answer to each problem (e.g., put a box around it).
- **Good luck!**

Problem 1 Find the extreme values of $f(x, y) = -x^2 - y$ subject to the constraint $x^2 + y^2 = 1$.

Problem 2 Estimate

$$I = \int_R\int e^{-x^2-y^2}\,dA$$

where $R = [0,1] \times [0,1]$. Use the midpoint rule with the following numbers of squares of equal size: 1 and 4.

Problem 3 Find the average value of $f(x,y) = x^2y$ over the rectangle with vertices (-2,-2), (-2,2), (2,-2), and (2,2).

Problem 4 In evaluating a double integral over a region D, a sum of iterated integrals was obtained as follows:

$$I = \int_D \int f(x,y)dA = \int_0^2 \int_0^{2y} f(x,y)dxdy + \int_2^3 \int_0^{12-4y} f(x,y)dxdy$$

Sketch the region D and express the double integral as an iterated integral with reversed order of integration.

Problem 5 Evaluate

$$I = \int_R \int \sqrt{x^2+y^2}dA$$

where $R = \{(x,y)|4 \leq x^2+y^2 \leq 16,\ y \geq 0\}$.

Problem 6 The joint density function for a pair of random variables X and Y is

$$f(x,y) = \begin{cases} Cx(1+y) & 0 \le x \le 1, 0 \le y \le 2 \\ 0 & otherwise \end{cases}$$

Write an integral whose value is $P(X+Y \le 1)$.

Problem 7 Set up (but do not evaluate) an integral to calculate the surface area of the part of the sphere $x^2 + y^2 + z^2 = a^2$ that lies within the cylinder $x^2 + y^2 = ax$ and above the xy-plane.

Problem 8 Sketch the solid whose volume is given by the iterated integral

$$\int_0^1 \int_0^{1-x} \int_0^{2-2z} dydzdx$$

Problem 9 Write an integral which, when evaluated, gives the volume of the solid that lies above the cone $\phi = \frac{\pi}{4}$ and below the sphere $\rho = 4\cos\phi$.