Material for this Department should be sent to Paul T. Mielke, Department of Mathematics, Wabash College, Crawfordsville, IN 47933, or to Shirley Hill, Department of Mathematics, University of Missouri, Kansas City, MO 64110.

PEAKS, RIDGE, PASSES, VALLEY AND PITS
A Slide Study of $f(x, y) = Ax^2 + By^2$

CLIFF LONG

The advent of computer graphics is making it possible to pay heed to the suggestion, "a picture is worth a thousand words." As students and teachers of mathematics we should become more aware of the variational approach to certain mathematical concepts, and consider presenting these concepts through a sequence of computer generated pictures.

To illustrate this notion, consider the following. In the study of functions of two variables it is usually emphasized that a regular non-planar point on a smooth surface can be classified as one of three distinct types: elliptic, parabolic, hyperbolic [1]. These may be illustrated using the functions $f(x, y) = Ax^2 + By^2$ with the origin being:

(a) elliptic if $A \cdot B > 0$;

(b) parabolic if $A \cdot B = 0$, $A \neq B$ (planar if $A = B = 0$);

(c) hyperbolic if $A \cdot B < 0$.

The slides reproduced here (see page 371) were made at Bowling Green State University from the screen of an Owens-Illinois plasma panel which is an output device for a Data General Nova 800 mini-computer. Many slide sets and super eight movies have been produced by college mathematics teachers under an NSF grant for "Computer Graphics for Learning Mathematics." The institute was held at Carleton College in Northfield, Minnesota, 55057, during the summers of 1973 and 1974. It was under the direction of Dr. Roger B. Kirchner, who, with no small amount of personal effort, has made these slides and movies available at minimum reproduction cost.

It must of course be kept in mind that while one good picture may be worth a thousand words, a thousand poorly chosen ones may be worthless.

Reference

1. D. Hilbert and S. Cohn-Vossen, Geometry and the Imagination, Chelsea, New York, 1952.

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THE TEACHING OF ELEMENTARY CALCULUS USING THE NONSTANDARD ANALYSIS APPROACH

KATHLEEN SULLIVAN

In the 1960's a mathematical logician, Abraham Robinson, found a way to make rigorous the intuitively attractive infinitesimal calculus of Newton and Leibniz, beginning a branch of mathematics called nonstandard analysis. When elementary calculus is developed from this nonstandard approach, the definitions of the basic concepts become simpler and the arguments more intuitive (see Robinson [2] or Keisler [1]). For example, the definition of the continuity of a function f at a point c is simply that x infinitely close to c implies that $f(x)$ is infinitely close to $f(c)$.

RIDGE

PASSES

VALLEY

PITS

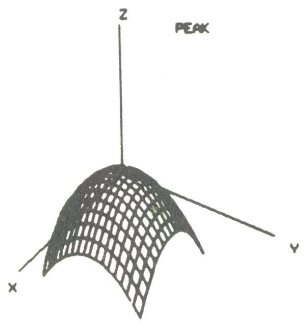
BY

CLIFF LONG

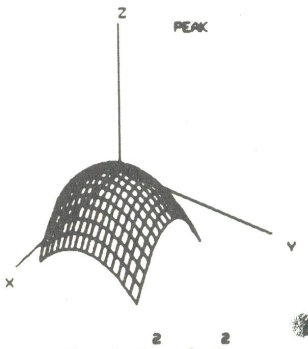
BGSU

A STUDY OF THE SURFACE

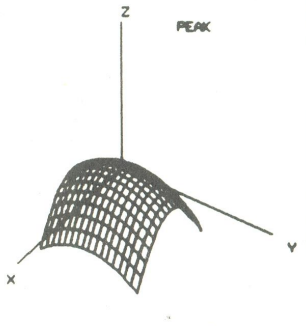
$Z = AX^2 + BY^2$



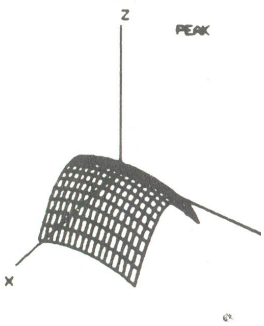
$Z = -1\ X^2 - .8\ Y^2$



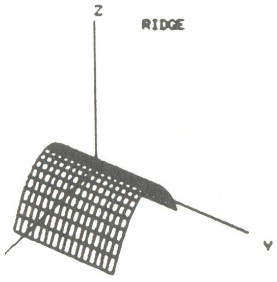
$Z = -1\ X^2 - .6\ Y^2$



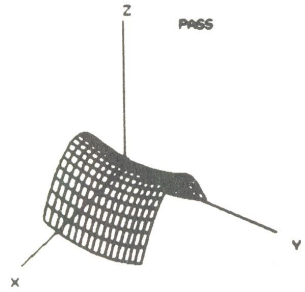
$Z = -1\ X^2 - .4\ Y^2$



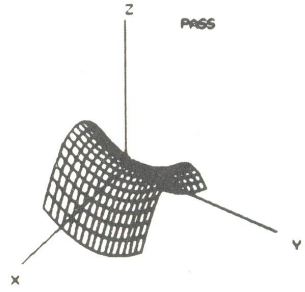
$Z = -1\ X^2 - .2\ Y^2$



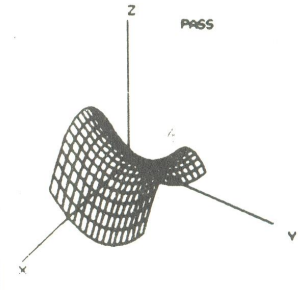
$Z = -1\ X^2 + 0\ Y^2$



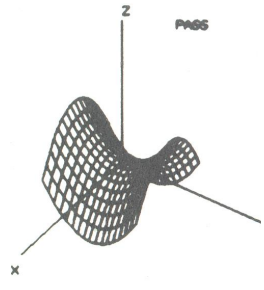
$Z = -1\ X^2 + .2\ Y^2$



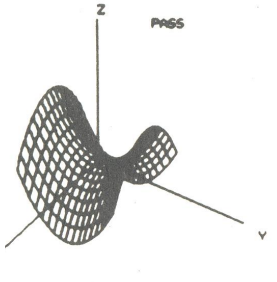
$Z = -1\ X^2 + .4\ Y^2$



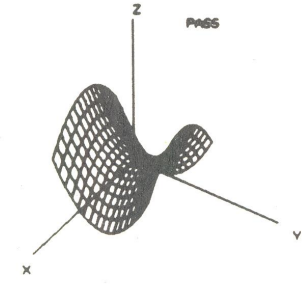
$Z = -1\ X^2 + .6\ Y^2$



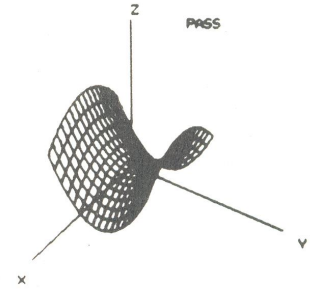
$Z = -1\ X^2 + .8\ Y^2$



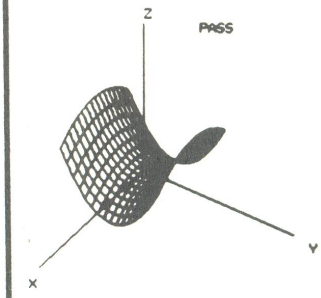
$Z = -1\ X^2 + 1\ Y^2$



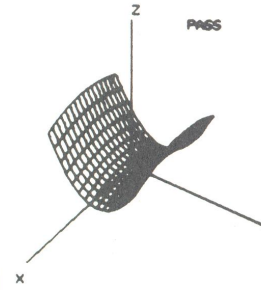
$Z = -.8\ X^2 + 1\ Y^2$



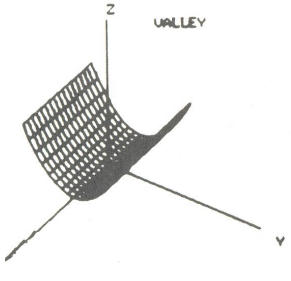
$Z = -.6\ X^2 + 1\ Y^2$



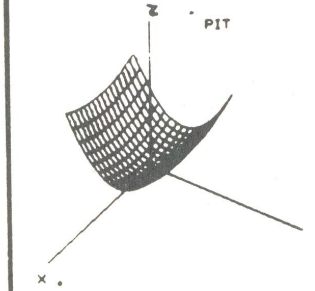
$Z = -.4\ X^2 + 1\ Y^2$



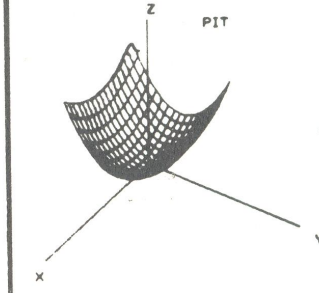
$Z = -.2\ X^2 + 1\ Y^2$



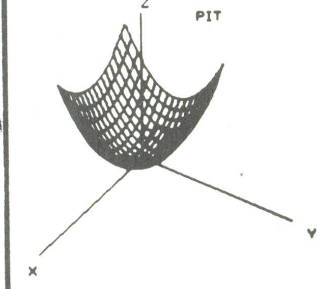
$Z = 0\ X^2 + 1\ Y^2$



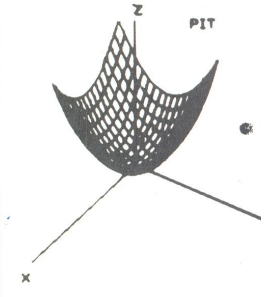
$Z = .2\ X^2 + 1\ Y^2$



$Z = .4\ X^2 + 1\ Y^2$



$Z = .6\ X^2 + 1\ Y^2$



$Z = .8\ X^2 + 1\ Y^2$