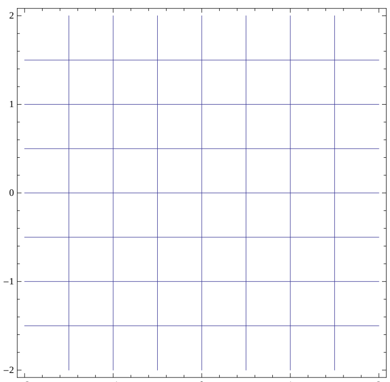
MAT329, Fall 2015: Chapter 16, plus old stuff

Name:

Directions: You **must** skip one entire problem – write "skip" on it clearly. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1: Consider the function $f(x, y) = \sqrt{x - y}$.

a. What is the domain of this real-valued function? Draw level curves (contour lines) of the function.



b. Compute the gradient of f. Use it to compute the directional derivative in the direction $\hat{\imath} + \hat{\jmath}$: why are you not surprised?

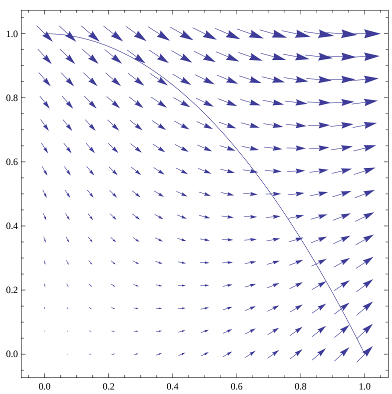
c. Use the limit definition of the derivative to compute f_x. You may need a common algebra trick! Justify any steps that aren't obvious.

Problem 2: Consider the vector-valued function $\mathbf{F} = \left\langle x + y^2, x^2 - y, 0 \right\rangle$.

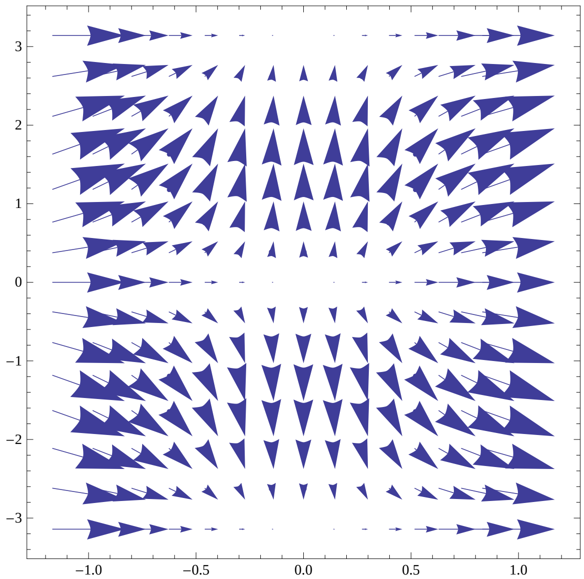
a. Compute the curl and the divergence of this field.

b. How do we know that $\mathbf{F}$ is **not** conservative?

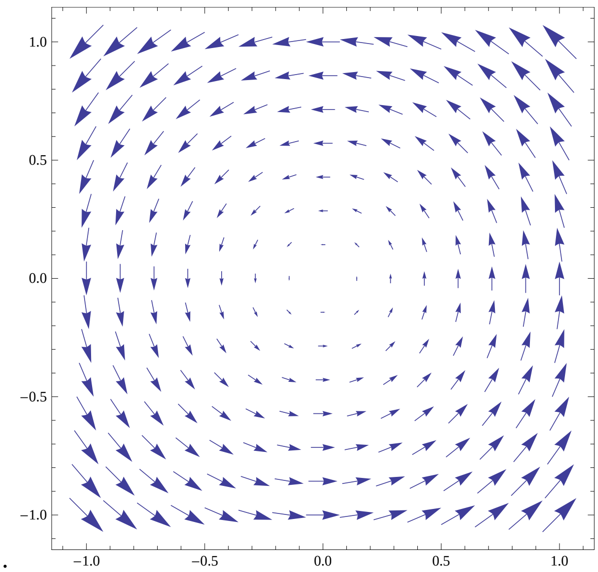
c. Compute the integral $\int_C \mathbf{F} \cdot \mathbf{dr}$, where C is the parabola $y = 1 - x^2$ shown in the vector field:



Problem 3: Consider the following two plots of vector fields of the form $\mathbf{F} = \langle P(x, y), Q(x, y)\rangle$:



A.



B.

Decide (if possible), **with reason(s)**,

a. whether either field is conservative.

b. whether either has non-zero curl.

c. whether either has zero divergence.

Problem 4: Let $A = \int\int dA$, the area of region D, where $0 \leq x \leq 1$ and $x^2 \leq y \leq x$.

a. Draw the region, and compute the integral directly as an area integral.

b. Now use Green's Theorem to evaluate this area using a line integral along the boundary of D, making an appropriate choice of Q and P.

Problem 5: Given $\mathbf{F} = \langle y(1-\sin(z)), x(1-\sin(z))+1, -xy\cos(z)\rangle$.

a. Show that this field is conservative.

b. What is $\nabla \mathbf{F}$?

c. Now do an infinite amount of work, by computing $\int_C \mathbf{F}\cdot d\mathbf{r}$ where C is any of all possible smooth paths from the origin to the point $(1, 1, \pi)$.

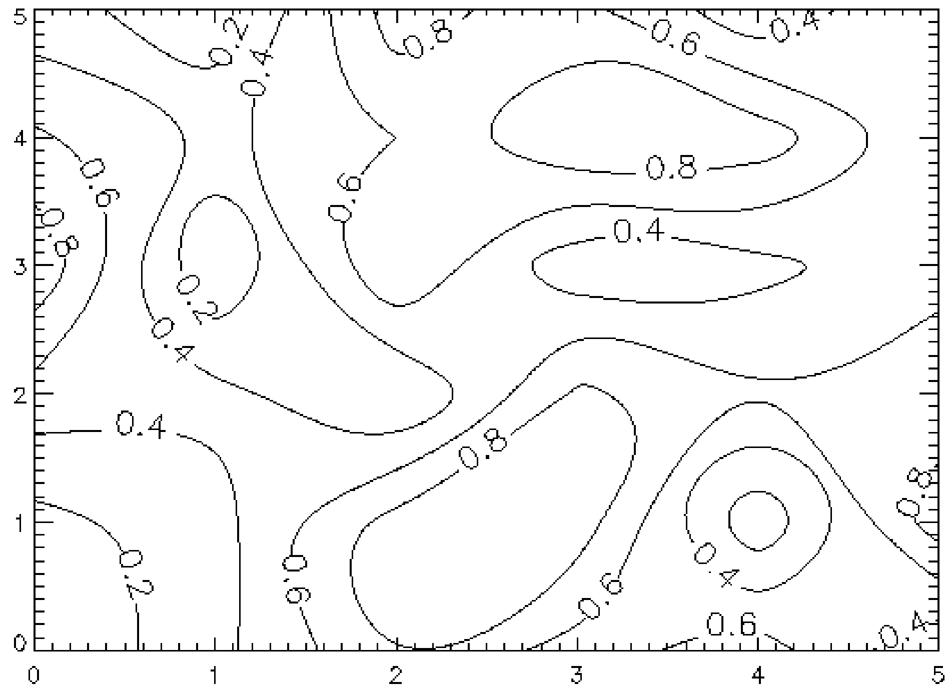
Problem 6: A sculpture carved from rock in the shape of a smooth mathematical surface is on the grounds of a university. Readings of the height of the sculpture (in feet) from the ground follow, taken along the square edge and at one point on the interior. The point $(x, y) = (0, 0)$ is at the bottom left corner (height 1), and the height 27 is at $(x, y) = (4, 4)$, with readings taken at 2 foot intervals in both the x and y directions: $\begin{matrix} 1 & 8 & 27 \\ 1 & 4 & 9 \\ 1 & 2 & 3 \end{matrix}$

a. Use our best method from class to estimate the volume of the sculpture. Explain your method.

b. Use the data to estimate the first partial derivative with respect to x at the point $(x, y) = (1, 1)$.

c. Use the data to estimate the mixed partial f_{xy} at the point $(x, y) = (1, 1)$.

Problem 7: Consider this contour map of a function $f(x, y)$:



a. draw about a dozen gradient vectors along the 0.6 contour that begins near (1.55,0). Illustrate changes in the **length** of the gradient vectors along the contour, as well as their directions. Explain choices to the right of the figure.

b. Indicate on the map likely locations of a maximum, a minimum and a saddle. Use M, m, and S to indicate these points. Show **just one of each type**, and justify your choices here.

c. What is the integral of $\oint_C \mathbf{F} \cdot \mathbf{dr}$, where C is the exterior of the graphic frame (the rectangular edge), positively oriented, and where $\mathbf{F}$ is the gradient vector field of the function f? Justify your answer.

Problem 8: Identify and classify any extrema of $f(x, y) = x(1 - y^2)$ on its domain.

Problem 9: For the same function, $f(x,y) = x(1-y^2)$, find and classify extrema subject to the constraint $x^2 + y^2 = 16$.

Problem 10: Sketch (and describe) the region given by the spherical coordinate constraints

$$0 \leq \phi \leq \frac{\pi}{2} \qquad\qquad 0 \leq \theta \leq \pi \qquad\qquad 1 \leq \rho \leq 2 (meters)$$

Do your best!:)

Now compute the mass M of this object, if its mass density is $m(\rho, \phi, \theta) = \frac{3}{\rho}$ (kg/m^3).

Problem 11. Compute the integral of $f(x, y) = xy - \sin(y) + y^3x^2$ over the unit circle by any method.