MAT329 Test 1: Chapter 14, Sections 1-5

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (10 pts). Consider $f(x, y) = 2 - 3x^2 - 2y$. Compute $f_x(x, y)$, using the limit definition of the partial derivative.

Problem 2 (10 pts). Consider the following snippet from a data set (with h and k positive step-sizes in the x and y directions, respectively):

	0	h	$2h$
0	9	12	13
k	5	10	15
$2k$	8	8	10

Approximate the following, justifying your approximation:

a. $f_x(h, k) \approx$

b. $f_y(h, k) \approx$

c. $f_{xx}(h, k) \approx$

d. $f_{xy}(h, k) \approx$

Problem 3 (20 pts). Consider $f(x, y) = \dfrac{\sin(x^2 - y^2)}{xy}$.

a. (4 pts) What is the domain of f?

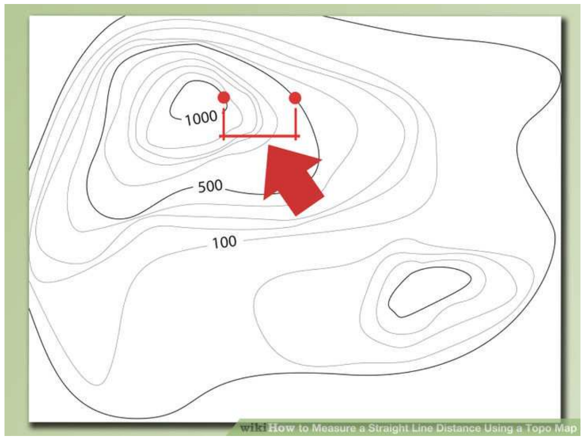
b. (4 pts) Suppose we define $f(0, 0) = 0$: determine whether f is continuous at the origin.

c. (4 pts) **Demonstrate** that the function f is differentiable at the point $(-1, 1)$.

d. (4 pts) Write the equation of the tangent plane to the function f at the point $(-1, 1)$.

e. (4 pts) The differential predicts the change in f as we move away from the point $(-1, 1)$. Use it to estimate $f(1.2, 1.3)$. Compare the differential dz to the increment Δz.

Problem 4 (10 pts). Consider the contour map following:



Assume that the contours are equally spaced, at 100 meter intervals. If the distance between the two points indicated is one kilometer, and the function values are elevations (in meters), compute or estimate the following (showing some work to justify your answer):

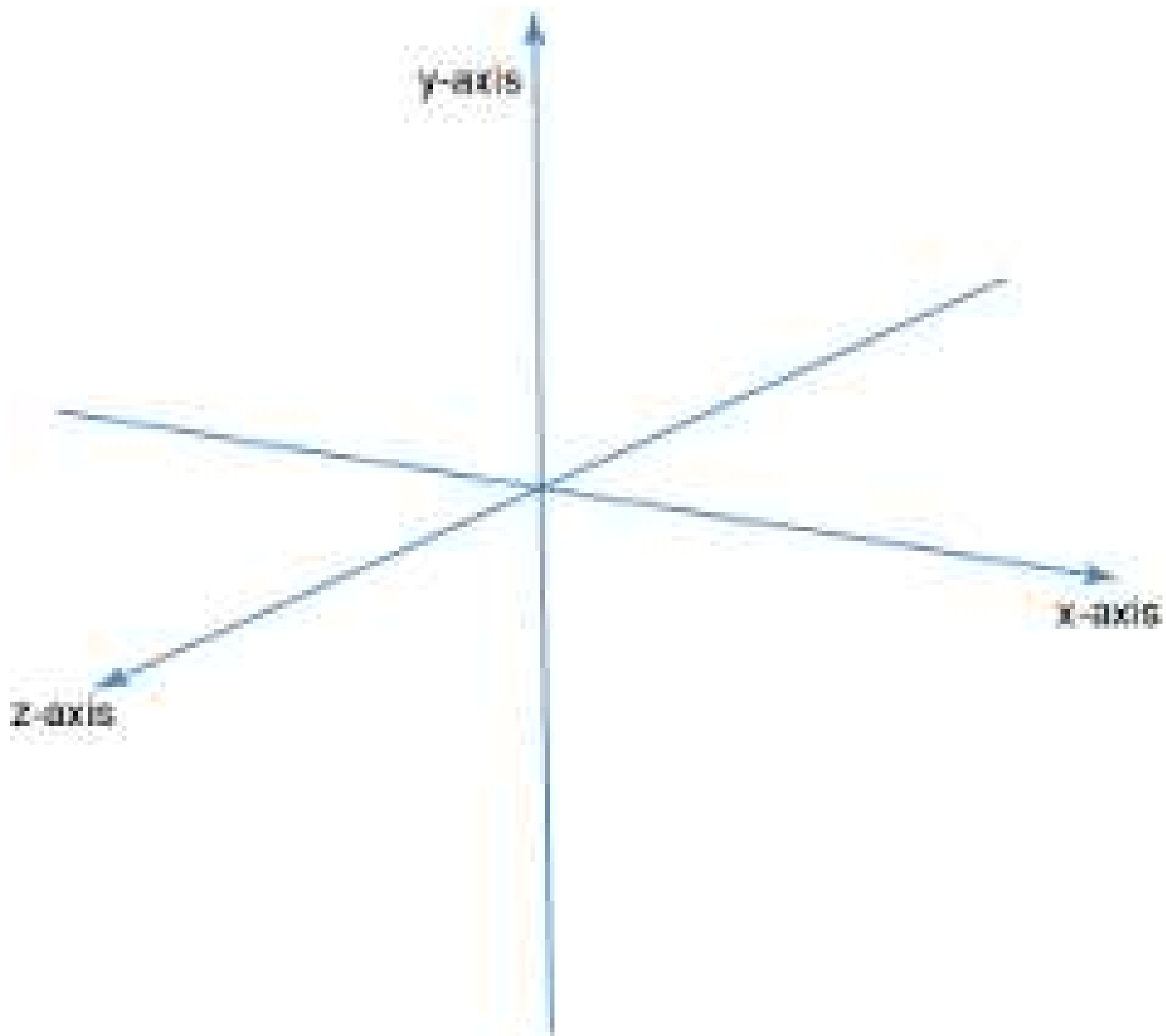
a. (4 pts) The average rate of change in elevation between the two points. Does this average rate provide a good approximation to the instantaneous rate of change there?

b. (3 pts) f_x at the point on the left.

c. (3 pts) f_y at the point on the left.

Problem 5 (10 pts).

a. Carefully graph the plane $L(x, y) = 2 - 3x + 2y$.



b. To the figure above add the points $(0, 1, 2)$, $(1, 2, -1)$, $(-1, 1, -2)$. Join these points with a plane passing through them.

Problem 6 (10 pts).

Consider the pressure in a balloon, modelled by the ideal gas law, where P is the pressure, V is the volume, T is the temperature, and c is a positive constant:

$$P(T, V) = c\,\frac{T}{V}$$

Assume that we steadily increase the volume at a rate of $\dfrac{1cm^3}{min}$, and that we steadily decrease the temperature at a rate of $\dfrac{0.1^oC}{min}$. How is the pressure changing in time when the balloon has a volume of $1m^3$, at a temperature of 36^oC?