MAT329 Test 2 (Spring 2016): Max/Mins, Lagrange, Integration

Name:

Directions: All problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. Let $f(x, y) = \sin(2x + 3y)$.

   a. (3 pts) Find the gradient of f.

   b. (3 pts) Evaluate the gradient at the point (-6,4).

   c. (4 pts) Find the rate of change of f at (-6,4) in the direction of the vector $\mathbf{u} = 2\hat{i} - \hat{j}$.

Problem 2. Find and classify the extrema of the function $f(x, y) = xy(1 - x - y)$. (What tricks can you use to make your life easier?)

Problem 3. Consider the function $f(x, y) = e^{xy}$. Use Lagrange multipliers to

a. (5 pts) write a system of equations that must be solved to find the maximum and minimum values of f subject to the constraint $x^3 + y^3 = 16$.

b. (5 pts) find the maxes and mins of f, subject to this constraint.

Problem 4. Estimate the volume of the solid that lies below the surface $z = xy(1 - x - y)$ and over the rectangle $R = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}$, using $n = 4$ equally sized sub-rectangles, and the

  a. midpoint method.

  b. trapezoidal method.

Problem 5. Let $f(x, y) = 2 - x - y$.

a. (4 pts) Sketch **the object** whose volume is given by $I = \int_0^2 \int_0^{2-x} f(x, y) \, dy \, dx$.

b. (6 pts) Compute I.

Problem 6.

a. (5 pts) Write the integral $I = \int_D \int x \, dA$ as an iterated integral, where D is the region bounded by the line $x = y - 1$ and the parabola $x^2 = 2y + 6$. Draw D first.

b. (5 pts) Evaluate I.