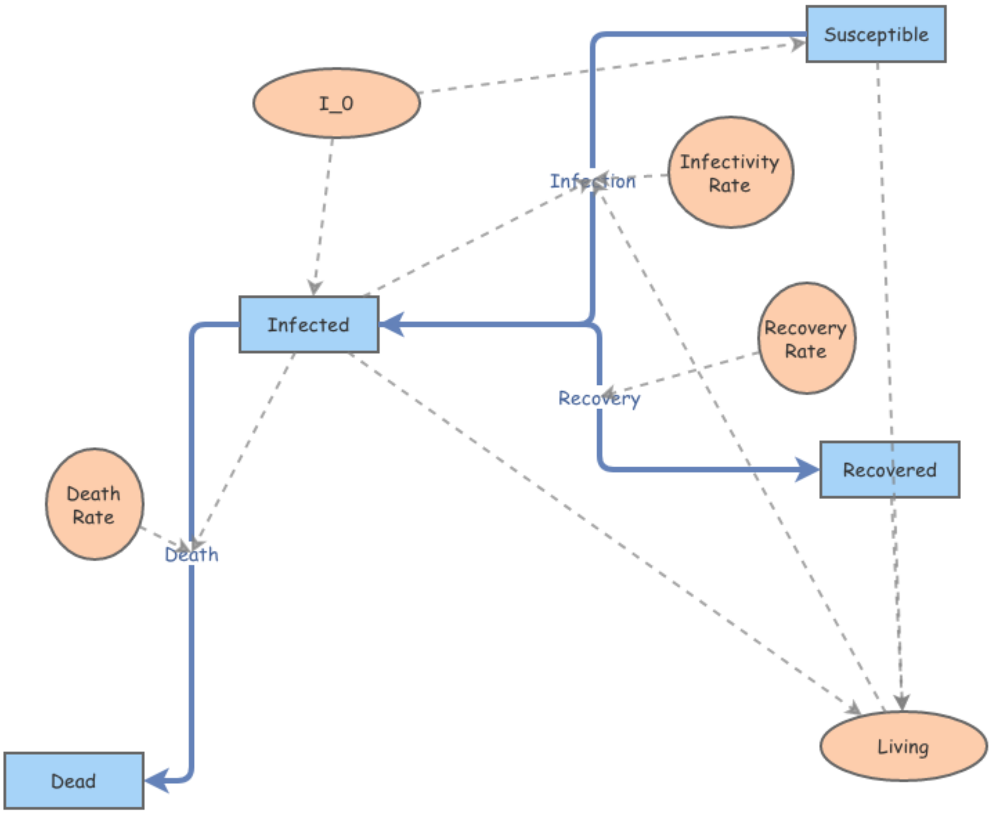
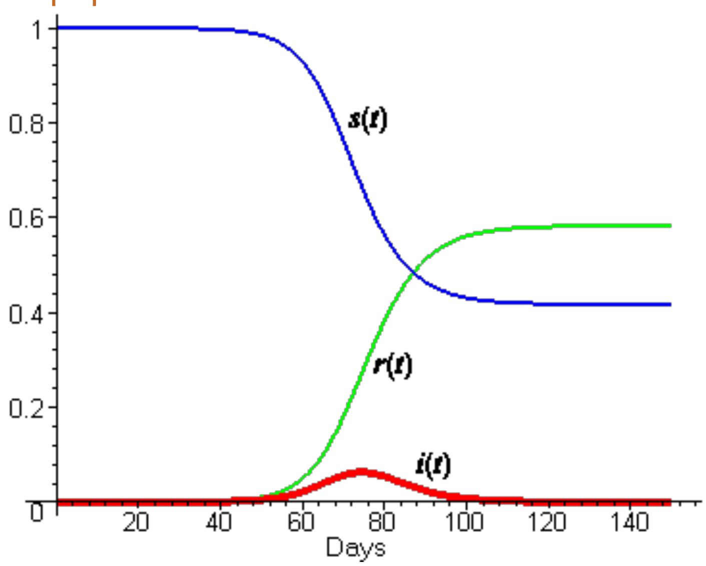
This is a simple SIR model, built to reproduce the one that we made in InsightMaker based on an MAA on-line SIR project:

https://www.maa.org/press/periodicals/loci/joma/the-sir-model-for-spread-of-disease-the-differential-equation-model

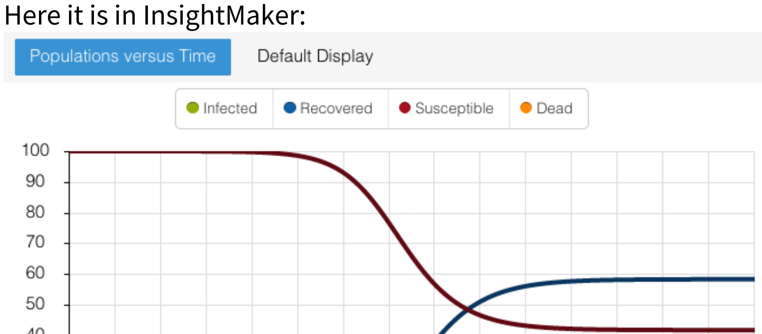
- Andy Long
  Spring, 2020



Starting parameters from the project lead to the following graphic from the MAA paper:



Here it is in InsightMaker:

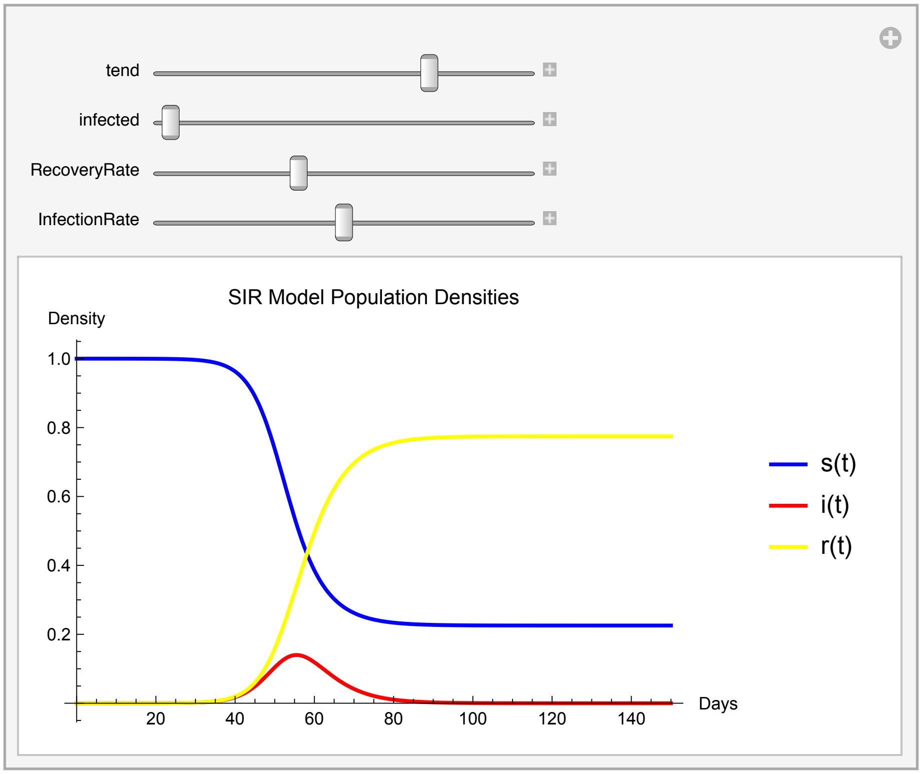


Looks like the MAA picture, pretty much. What's happening is that there is a long pause, during which the infection is growing, and then it bursts forth, big growth in recovered, susceptible drops, and then then infected population dies out. There are no longer enough susceptibles to support an epidemic (sort of a herd immunity), so as t -> infinity the populations of the three will stay stable at these values.

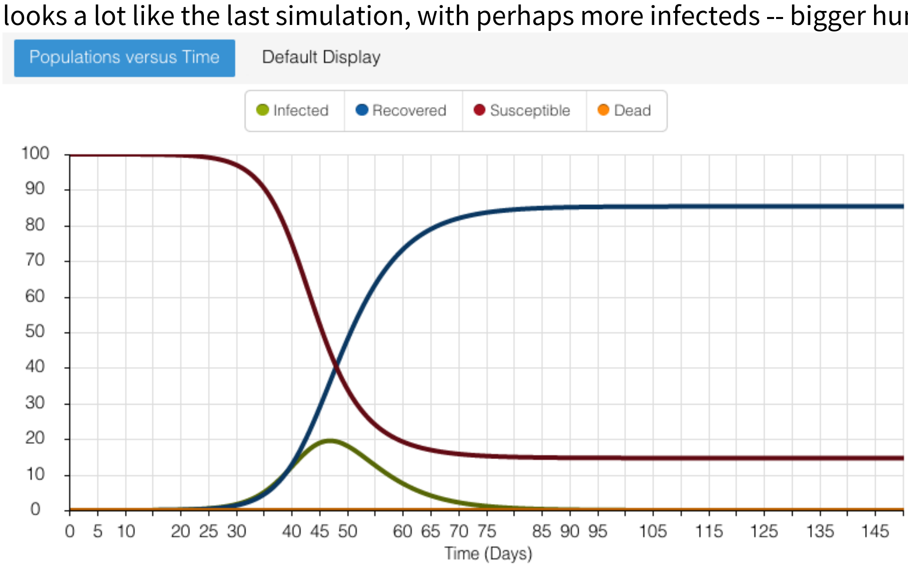
In[20]:=
```
Clear["Global`*"]
```

In[21]:=
```
Manipulate[
 lv =
  NDSolve[
   {(*  susceptible: *)
    sus'[t] == -InfectionRate sus[t] inf[t],
    (* infected: *)
    inf'[t] == InfectionRate sus[t] inf[t] - RecoveryRate inf[t],
    (* recovered: *)
    rec'[t] == RecoveryRate inf[t],
    sus[0] == 1 - infected,
    inf[0] == infected,
    rec[0] == 0
   },
   {sus[t], inf[t], rec[t]},
   {t, 0, tend}];
 Plot[Evaluate[{sus[t], inf[t], rec[t]} /. lv], {t, 0, tend},
  PlotRange → All,
  AxesOrigin → {0, 0}, PlotStyle → {{Blue, Thick}, {Red, Thick}, {Yellow, Thick}},
  PlotLabel → "SIR Model Population Densities",
  AxesLabel → {"Days", "Density"},
  PlotLegends → {"s(t)", "i(t)", "r(t)"}
 ],
 {{lv, {}}, None},
 {{tend, 150}, 2, 200, 10},
 {{infected, 1.27/1000000}, 0, 1, .001},
 {{RecoveryRate, 1/3}, 0.0, .7, .01},
 {{InfectionRate, 0.5}, 0, 1, .001}
]
```

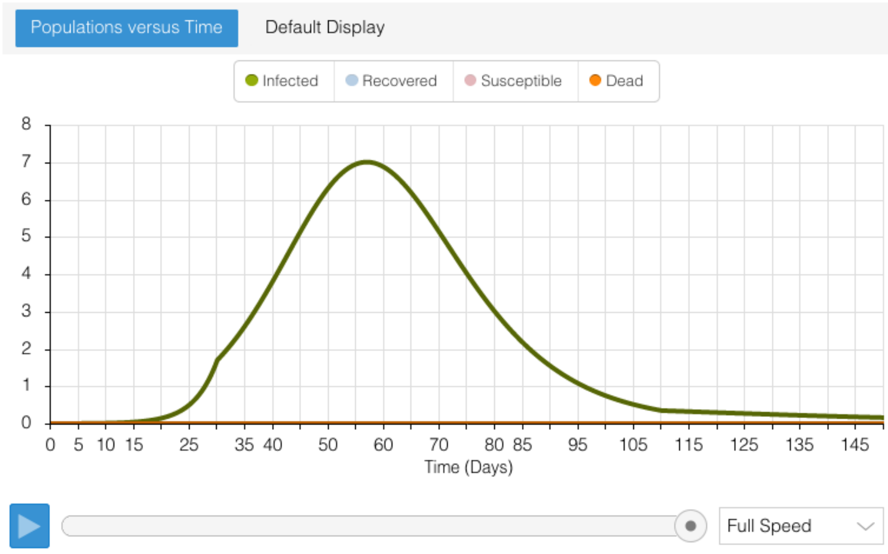
Out[21]=

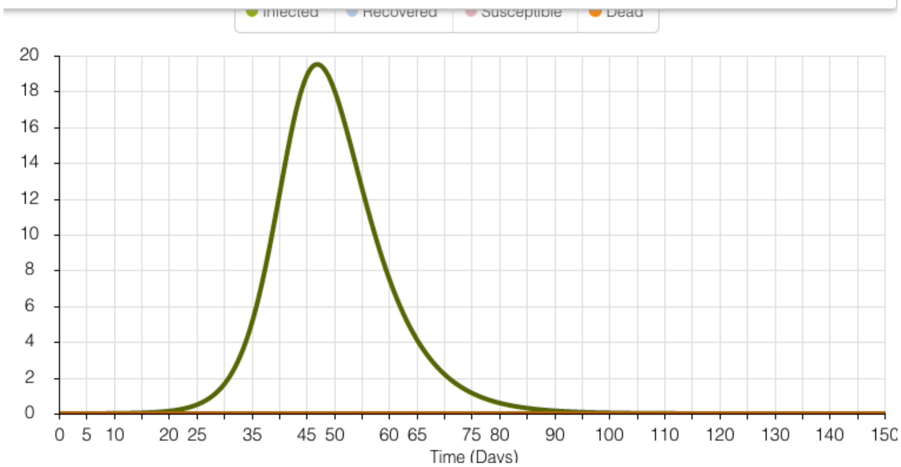


Simulation with these new parameters:
looks a lot like the last simulation, with perhaps more infecteds -- bigger hump, and less spread.

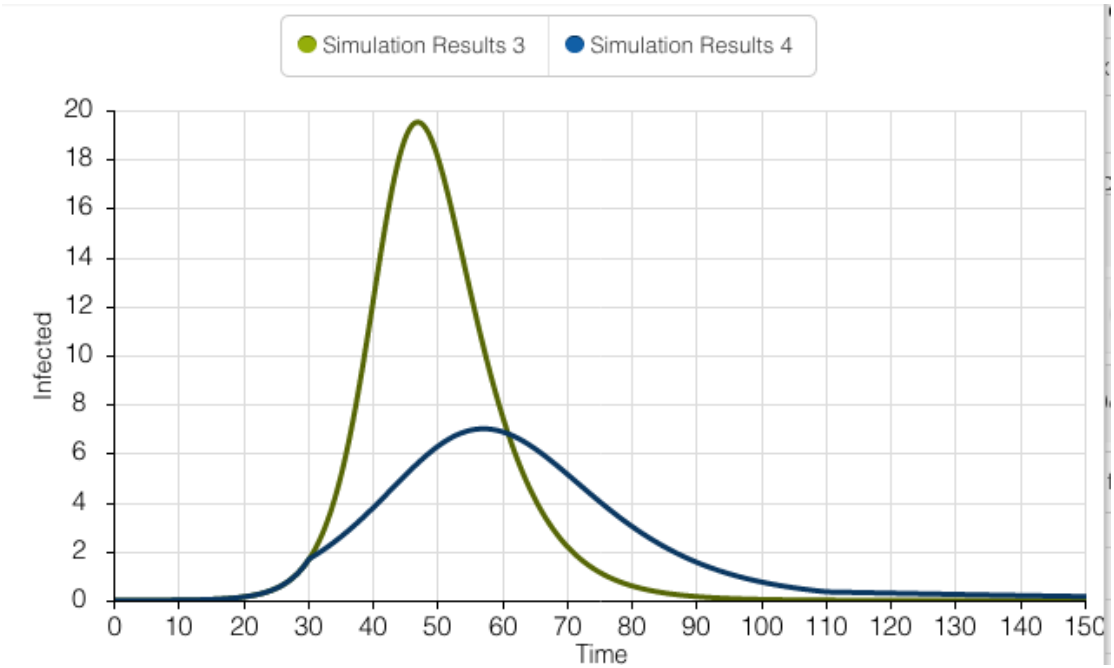


Now that we’ve implemented the step function, the infecteds have some little non-differentiable spots. The infection isn’t as big, either.

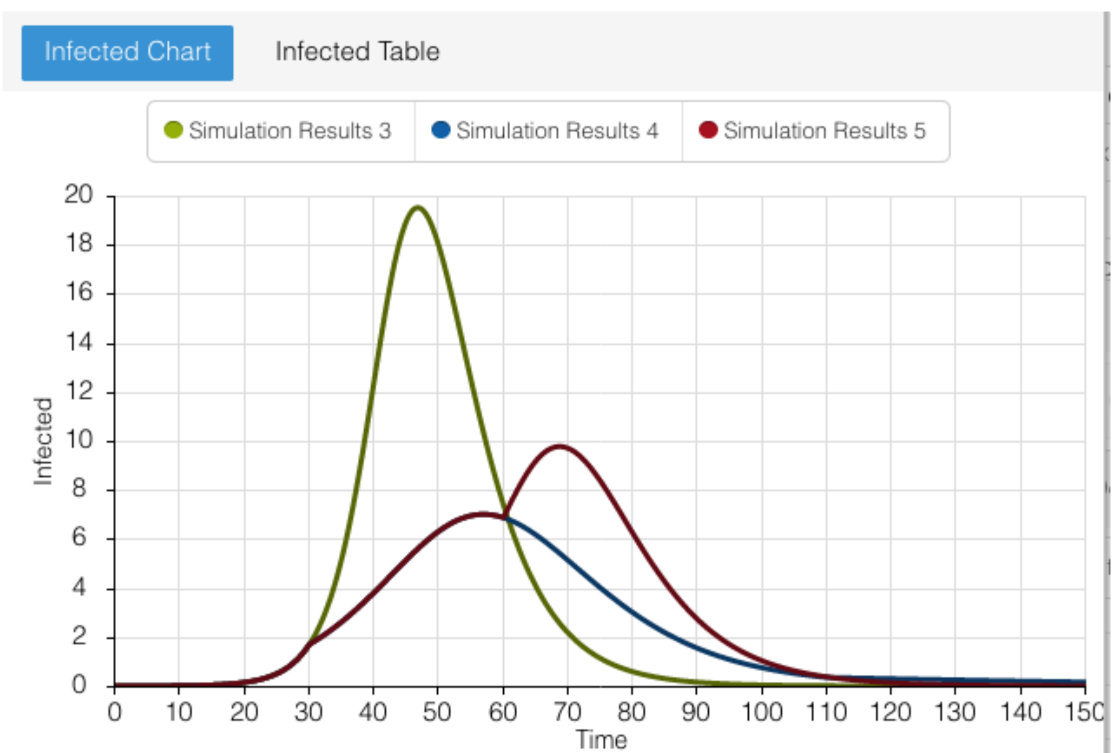




We’ve flattened the curve, it seems:



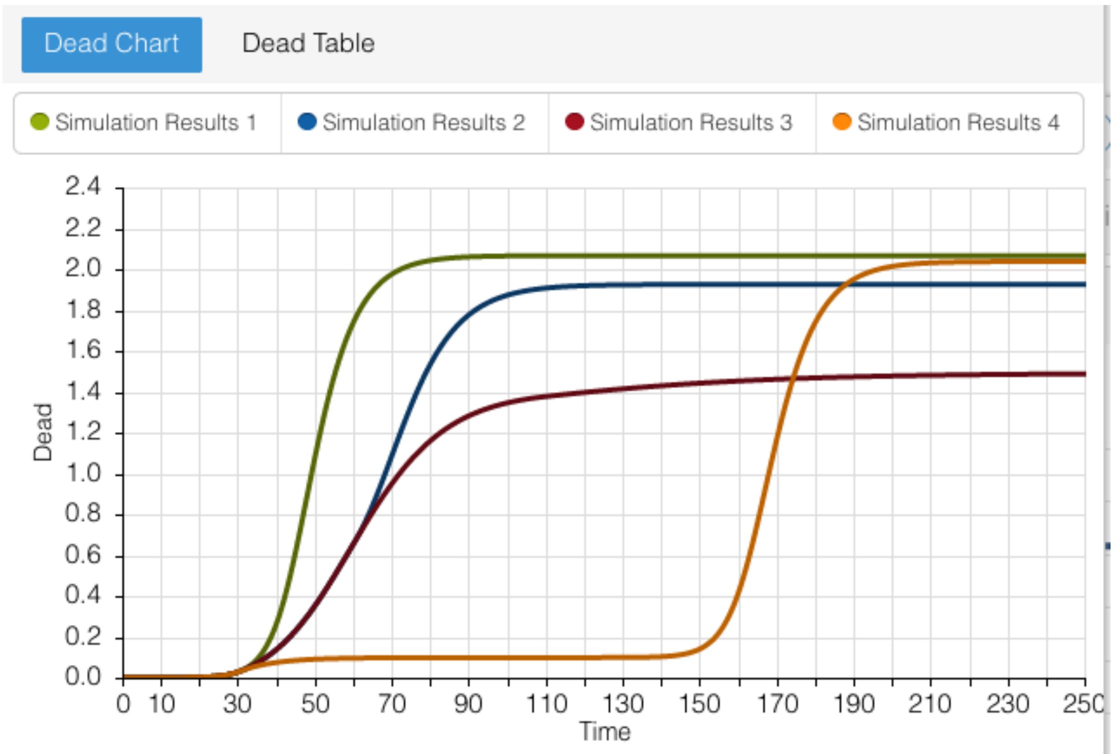
We had flattened it, but by releasing too soon, the infection flared up again: we release it at 60 days, when before we'd kept it locked down for 110 days. Those extra days kept the peak lower, and so we didn't overwhelm our health care facilities.



Create graphs using successively more vigorous social distancing (why is this succession of α values represent more vigorous social distancing?). Set your time setting for days from 0 to 250.

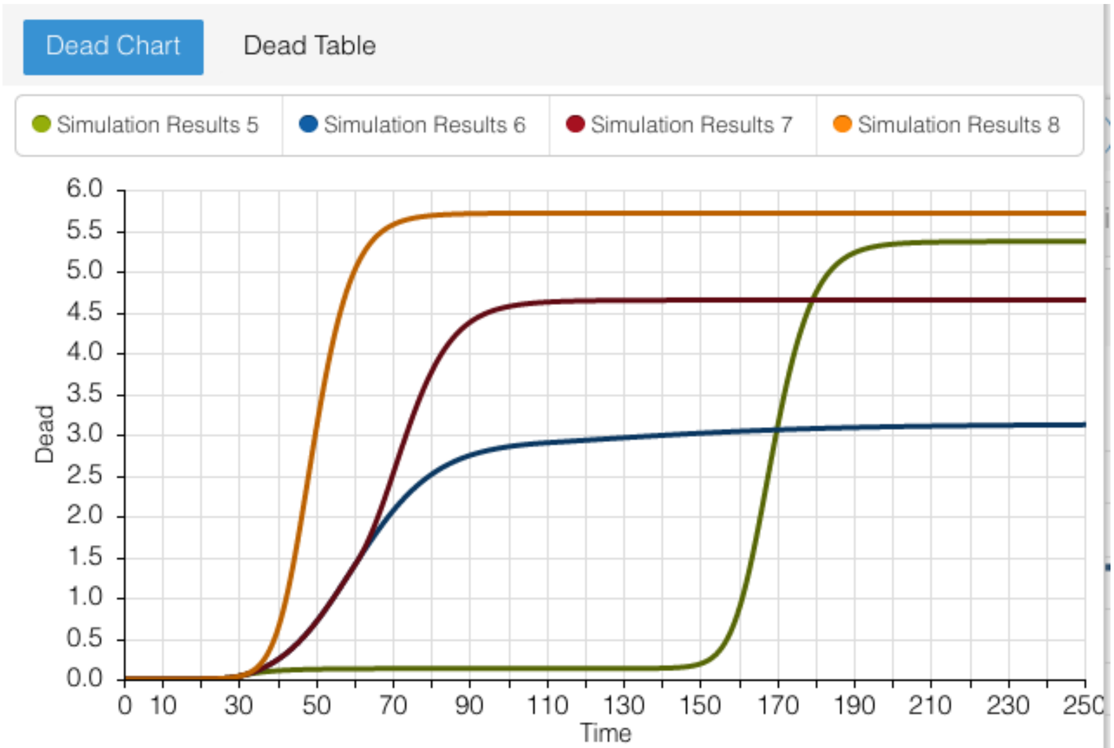
0.45
IfThenElse(Time()<{30 days},0.45,IfThenElse(Time()<{60 days},.3,.45))
IfThenElse(Time()<{30 days},0.45,IfThenElse(Time()<{110 days},.3,.45))
IfThenElse(Time()<{30 days},0.45,IfThenElse(Time()<{110 days},.1,.45))

Compare the number of dead under these conditions:



The surprise is that the dead in the fourth end up surpassing the dead in what seemed less stringent conditions. We suppressed the epidemic early, only to have it go “underground” for awhile -- then it burst onto the scene later, when the population’s guard had been let down.

Finally let’s make a change in the death rate. Rather than being a fixed constant of the infected population, it seems that, should the infected population become too high, the hospital systems will be overwhelmed, and the death rate will skyrocket. Hence it makes sense to make δ a function of the infected population. So create a link from the infected population to the death rate δ, and set δ=.005*(1+3I/(5+I))



The obvious thing is that the number of deaths have skyrocketed; but it is basically the same in that what seemed like the best situation -- slapping down the virus for a long time, suppressing it very strongly, ended up costing more lives in the end.

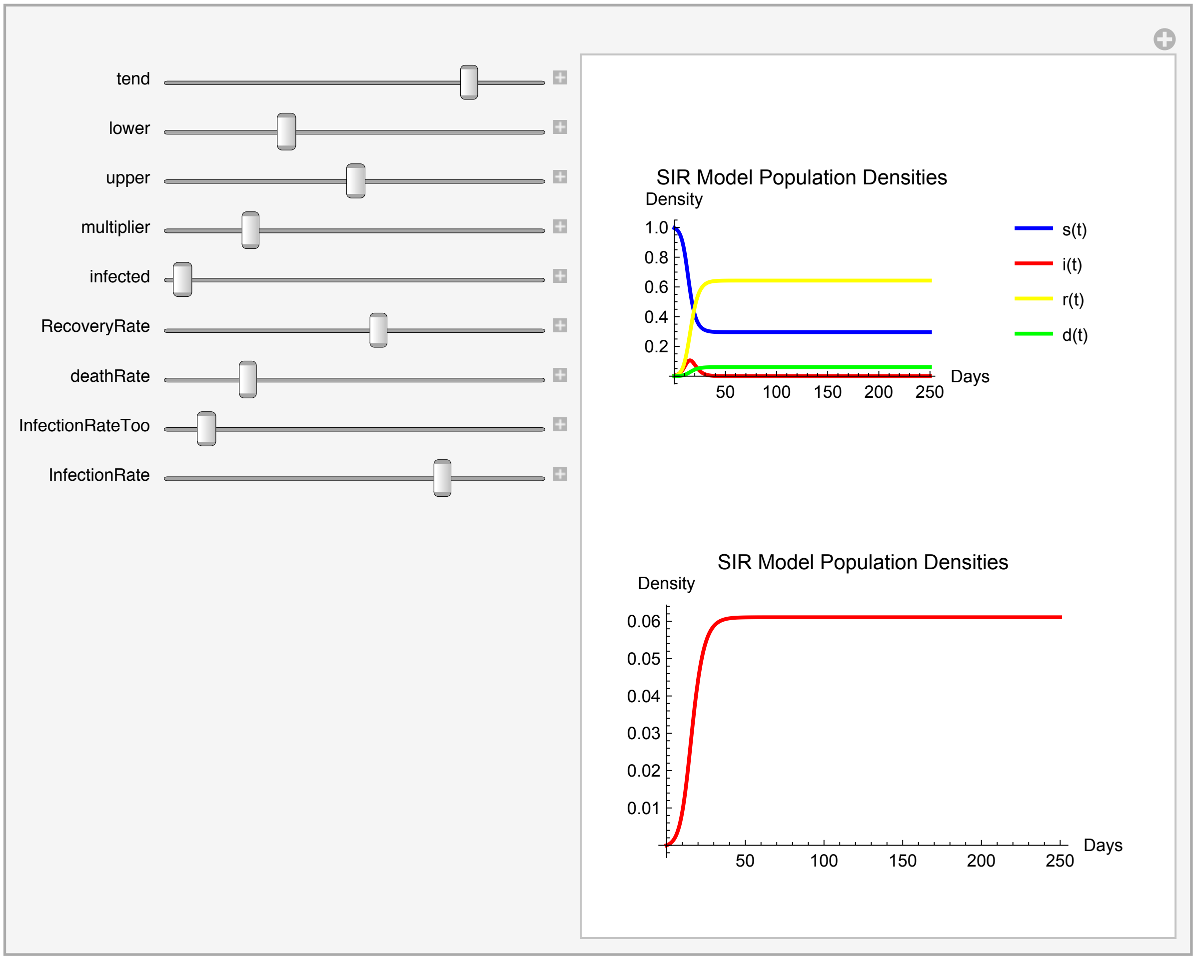
We have to let the disease "run its course", in a sense; but the best strategy seemed to be a long lock-down, but with some disease to turn susceptibles into recovered, and give some herd immunity. Hard news!

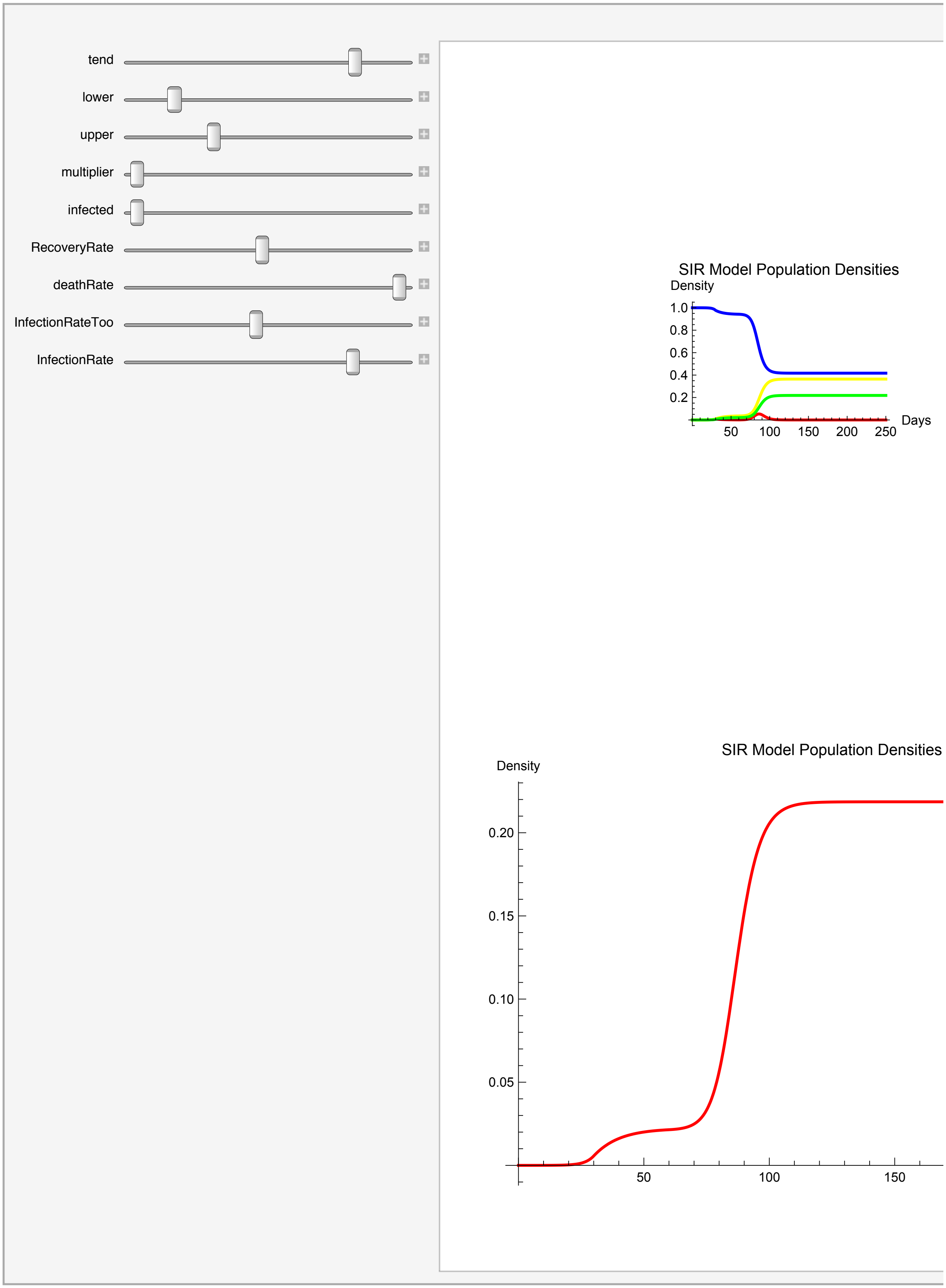
But we have to be careful not to overrun our health care facilities.

In[28]:=
```
Manipulate[
 lv =
  NDSolve[
   {
    sus'[t] ==
     - If[t ≤ lower || t ≥ upper, InfectionRate, InfectionRateToo] sus[t] inf[t],
    inf'[t] == If[t ≤ lower || t ≥ upper, InfectionRate, InfectionRateToo]
        sus[t] inf[t]
      - RecoveryRate inf[t]
      - deathRate inf[t], (* * (1+multiplier*inf[t]/(3+inf[t])) *)
    rec'[t] == RecoveryRate inf[t],
    dead'[t] == deathRate inf[t],
    sus[0] == 1 - infected,
    inf[0] == infected,
```

```
     rec[0] == 0,
     dead[0] == 0
    },
    {sus[t], inf[t], rec[t], dead[t]},
    {t, 0, tend}];
  GraphicsGrid[
   {
    {Plot[Evaluate[{sus[t], inf[t], rec[t], dead[t]} /. lv], {t, 0, tend},
      AxesOrigin → {0, 0},
      PlotStyle → {{Blue, Thick}, {Red, Thick}, {Yellow, Thick}, {Green, Thick}},
      PlotLabel → "SIR Model Population Densities",
      AxesLabel → {"Days", "Density"},
      PlotLegends → {"s(t)", "i(t)", "r(t)", "d(t)"}
     ]},
    {
     Plot[Evaluate[{dead[t]} /. lv], {t, 0, tend},
      AxesOrigin → {0, 0}, PlotStyle → {{Red, Thick}},
      PlotLabel → "SIR Model Population Densities",
      AxesLabel → {"Days", "Density"}
     ]
    }}
  ],
  {{lv, {}}, None},
  {{tend, 250}, 2, 300, 10},
  {{lower, 30}, 2, 200, 10},
  {{upper, 60}, 2, 200, 10},
  {{multiplier, 0}, 0, 5, 1},
  {{infected, 1.27/1000000}, 0, 1, .001},
  {{RecoveryRate, 1/3}, 0.0, .7, .01},
  {{deathRate, .1}, 0.0, .2, .001},
  {{InfectionRateToo, 0.5}, 0, 1, .001},
  {{InfectionRate, 0.9}, 0, 1, .001}
 ]
```

Out[28]=


tend
lower
upper
multiplier
infected
RecoveryRate
deathRate
InfectionRateToo
InfectionRate
SIR Model Population Densities
Density
1.0
0.8
0.6
0.4
0.2
50
100
150
200
250
Days
s(t)
i(t)
r(t)
d(t)
SIR Model Population Densities
Density
0.06
0.05
0.04
0.03
0.02
0.01
50
100
150
200
250
Days


tend
lower
upper
multiplier
infected
RecoveryRate
deathRate
InfectionRateToo
InfectionRate
SIR Model Population Densities
Density
1.0
0.8
0.6
0.4
0.2
50
100
150
200
250
Days
SIR Model Population Densities
Density
0.20
0.15
0.10
0.05
50
100
150