A Stick Found in the Woods

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Abstract

We have a stick, appearing to be a tool of measurement, but free of: instructions, context, and easily discernible units of measure. Our task is to use modeling to find some relationship between the different markets on this stick. Using the methods used in this, and other classes, we will attempt to do so.

1 Plan

1.1 Data Introduction

Plainly, our data set comes from the stick itself. The one unit of measure we can knowingly take are the inch marks, from 2 to 30, on both sides of the tool. Being known, this is what we used for our x-axis. Along both sides, as well as both edges, there are additional numbers that do not follow units of measure we are familiar with for calculating size. Each side contains two sets of "other" numbers, running across the top and bottom.

The first step in our plan is simply to put these numbers into a table. With inches as our x-axis, we have a total of six possible y-axis: both sides' top and bottom values, as well as the values on the edges. Plotting this data is a natural second step. However, there are six possible plots. Before plotting this data, we decided to research this tool, hoping to gain insight on its markings.

1.2 Research Opportunity

Fortunately, we were given a sliver of context found on the handle. Just legible enough are etched the words "Lufkin Log Ruler," and "ONT, Canada," short for Ontario, Canada. That allowed us an avenue in which we could decide which data set of values we wanted to use.

1.3 Data Concerns

A concern exists with the over, or under, compilation of data. Do the values that run along the top of each side differ in scale from those on the bottom? Are they part of the same scale? Then there is the issue of the marks on each edge? They are not uniform, and there is no clear indication if one edge is to be used in conjunction with a specific side. Thus, the concern of becoming lost in the woods is an issue.

2 Research

2.1 The Ontario Rule

"Lufkin Log Rule," and "Ontario," were crucial terms to search. Sifting through many documents lead us to the following: "The measurement of forest products in Canada: Past, Present, and Future," written by J.W. Ker in the Forestry Chronicle. The exhaustive article explained the history of log rules - methods for determining the board feet (FBM) of a tree or log - before milling timber. He describes the history of those rules in every Canadian province, including Ontario.

The east-central Candian province has had a few log rules as its standard; from the Doyle Rule, to the Scribner rule, and finally the Ontario Rule in 1953. That gave three leads to pursue, but the aptly named Ontario Rule prompted us to start there. The article provided the following chart:

TABLE 1

BOARD-FOOT CONTENT OF 16-FOOT SAWLOGS AS ESTIMATED BY VARIOUS LOG RULES

Top D.I.B., in.	Log Rule Volume, board feet								
	Doyle	Ont.	B.C.	Scribner Decimal C	Scribner Formula Rule	Alberta Modified Int'l 5/16"	Int'l 5/16" Rule Rounded	Quebec (Roy)	N.B.
4	0	5	5				5	7	9
6	4	17	15	20	12	18	20	20	20
8	16	34	32	30	31	37	40	39	40
10	36	57	54	60	55	62	65	65	64
12	64	86	84	80	86	93	95	97	96
14	100	121	118	110	123	129	135	135	130
16	144	162	160	160	166	172	180	180	170
18	196	209	207	210	216	222	230	231	229
20	256	261	261	280	272	277	290	289	300
22	324	320	319	330	334	338	355	353	362
24	400	384	384	400	403	405	425	423	432
26	484	454	455	500	478	479	500	500	507
28	576	530	531	580	559	558	585	583	614
30	676	612	611	660	647	644	675	673	706

From here, we were given a clue as to what the unknown stick is used for. The inch markings are used to measure the diameter of the log. Then, depending on the log rule you are using, the corresponding value is the board feet per 16 feet of timber length.

Table: 1 Volume Table (Ontario Rule)
Volume in Board Feet

Diameter in Inches Inside Bark at Small End of Log	Length of Log (in feet)				
	8'	10'	12'	14'	16'
4	3	3	4	5	5
5	5	6	8	9	10
6	8	10	13	15	17
7	12	15	19	22	25
8	17	21	26	30	34
9	22	28	34	39	45
10	29	36	43	50	57

After further research, the true use of the "Stick found in the woods" became known. As previously mentioned, it was determined that the stick is used for measuring lumber products in Canada (and sometimes America), following the Ontario rule. After finding this chart provided by the Ontario Woodlot Association, it was found that the stick uses the Ontario Log Rule to calculate the volume of a log following the equation FBM $= (0.55 * D^2 - 1.2 * D) * L/12$, where L is the length of log measured in feet, D is the top diameter of the log measured in inches, and FBM is called the board feet of a log, a specific unit of measure $(12" * 12" * 1")$ used for measuring the volume of lumber.

It is important to make the distinction that a log and the height of a tree for milling are different. A log is the division of a tree, and as an example, a mill using a 12 foot log standard will cross-cut the timber every 12 feet.

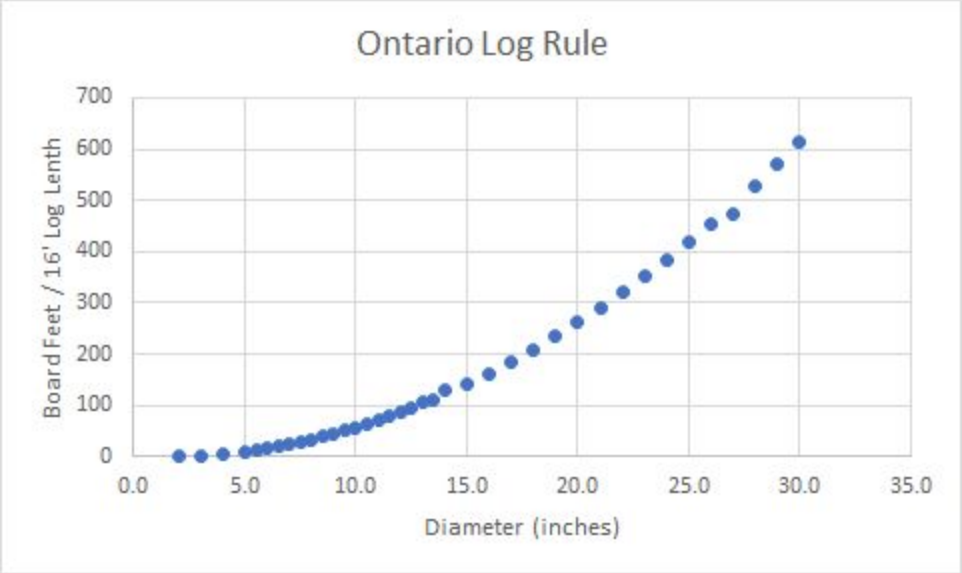
After comparing the provided chart with the unknown stick, it was easy to discern that the stick is used to find the Length of Log based on the diameter of the

inside bark at the small end of a log. Our stick includes all of the information on the provided graph except for the column labeled 8'.

3 Calculations

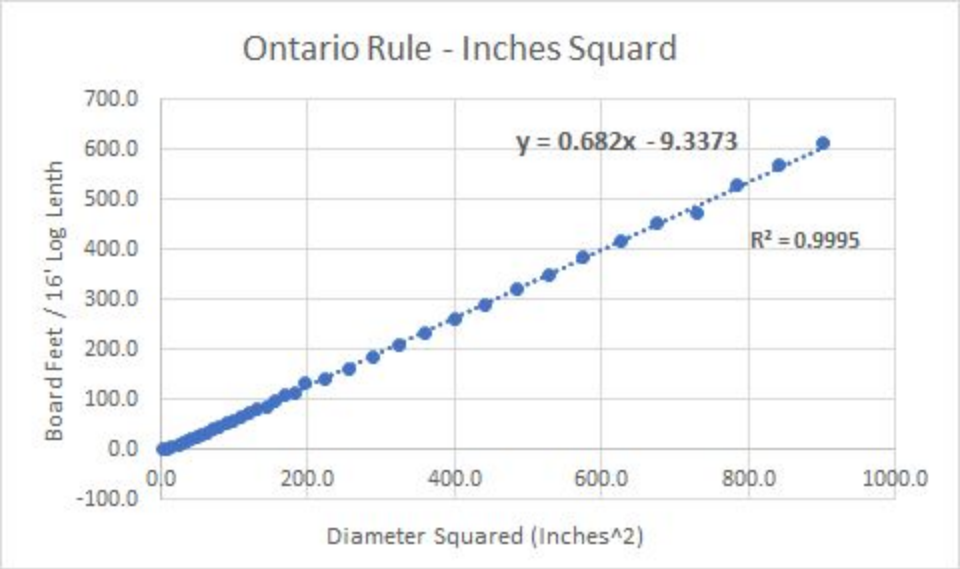
3.1 Plot and Best Fit

To start our data plotting, we chose the 16 foot log standard. Those plots were made in the graph below.



"Don't fit a line to a banana." - a wise man

Plotting the inches against the board feet (see graph above for reference), it was determined that the plotted data resembled a bell shaped curve. Because of this, a different approach had to be taken to produce a good linear regression of the data. The first inclination was to square the x-values to produce a new graph (graph #2). After plotting the squared diameter against the board feet, a linear regression was procured, with an R^2 value of 0.9995, resulting in an accurate progression and confirming our compiled data.



3.2 Further Plots

Similar plots and regressions were done for the 10, 12, and 14 foot log standards. They are displayed at the end of our paper.

4 Conclusion

The duality of our research and successful linear regressions led us to the conclusion that we can recreate the values on the log ruler. To do so, we would use the following, where D is diameter in inches, and H height in feet - of the tree, not the log. These functions are not simple cylinder volumes for many reasons. The functions here account for interior and exterior defects of the log, which lowers the effective volume of the log for its intended purpose.

$$Using\ 10'\ log \rightarrow BFM = (0.4301D^2 - 7.3983)$$
$$Using\ 12'\ log \rightarrow BFM = (0.5164D^2 - 8.4599)$$
$$Using\ 14'\ log \rightarrow BFM = (0.6074D^2 - 15.043)$$
$$Using\ 16'\ log \rightarrow BFM = (0.682D^2 - 9.3373)$$

Those values can be confirmed against the Ontario Log Rule formula, given in section 2.1. As well, it is important to clarify that the lower values for diameter were the least accurate in the regression, meaning the smaller the diameter input into the regression equation, the less accurate the output.

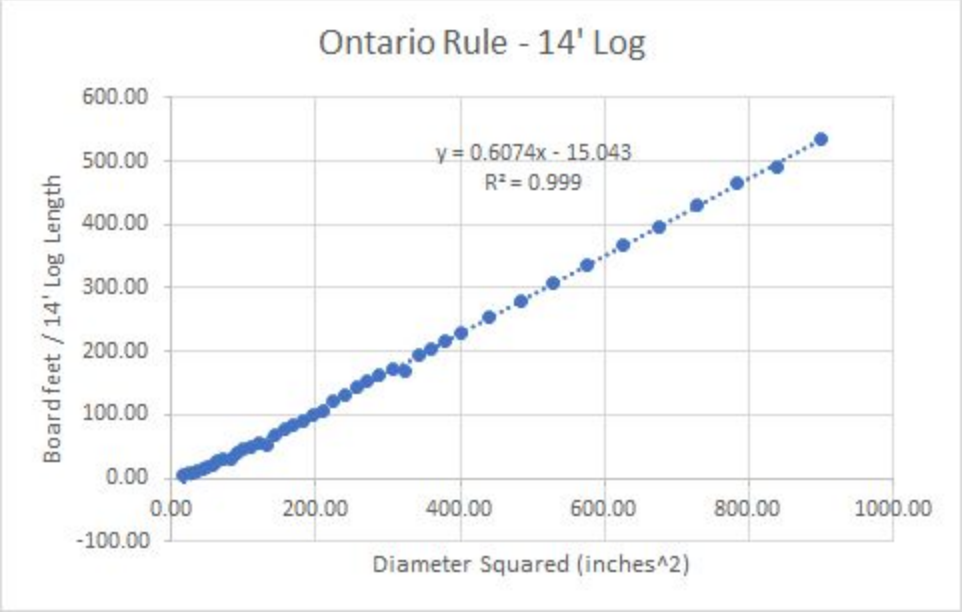
5 References

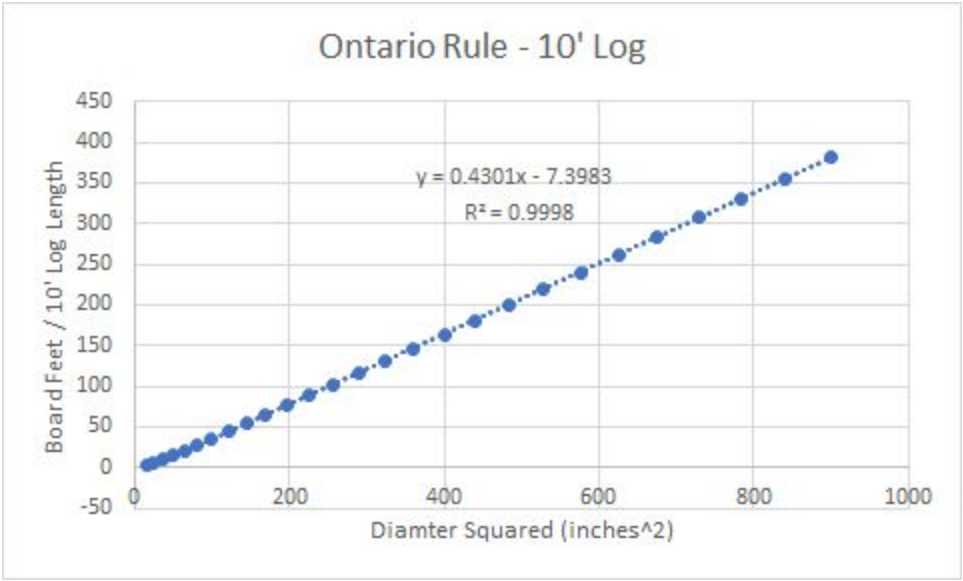
Log Scaling - Ontario Log Rule. Ontario Woodlot Association , 2003.

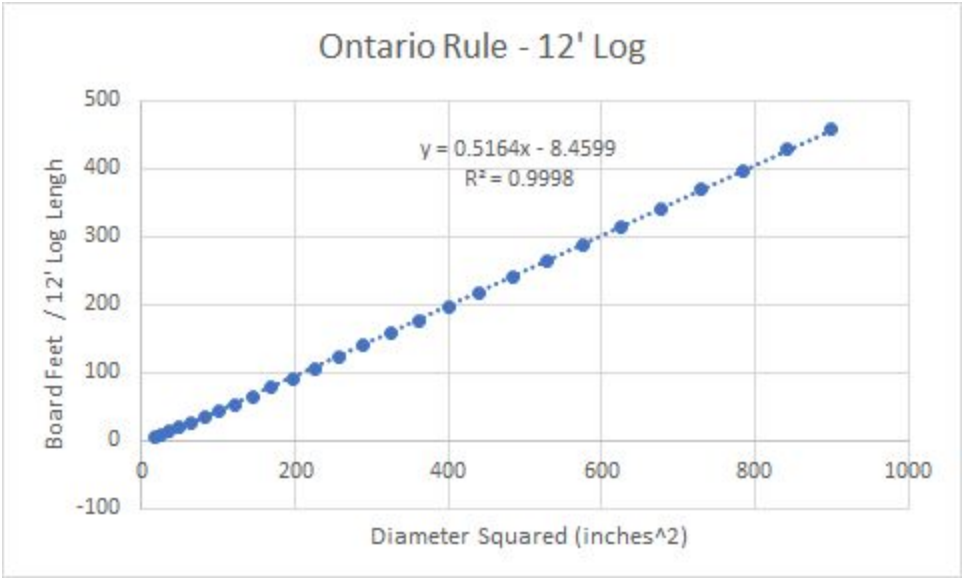
KER, J W. *HE MEASUREMENT OF FOREST PRODUCTS IN CANADA: PAST, PRESENT AND FUTURE HlSTORlCAL AND LEGlSLATlVE BACKGROUND*. The Forestry Chronicle, 28 Jan. 2020, pubs.cif-ifc.org/doi/pdf/10.5558/tfc42029-1.

"Tree Measurement." *YouTube*, YouTube, www.youtube.com/watch?v=R8kzuq6ecKI.

6 Further Graphs




Ontario Rule - 10' Log
y = 0.4301x - 7.3983
R² = 0.9998
Board Feet / 10' Log Length
Diamter Squared (inches^2)
450
400
350
300
250
200
150
100
50
0
-50
0
200
400
600
800
1000


Ontario Rule - 12' Log
y = 0.5164x - 8.4599
R² = 0.9998
Board Feet / 12' Log Lengh
Diameter Squared (inches^2)
500
400
300
200
100
0
-100
0
200
400
600
800
1000