Epic Cross Country Battles: Thad or Thomas – Who's the GOAT?

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Abstract

Father and son, successful cross country runners from different eras, each makes a reasonable argument to have been the better runner "in his prime". The resolution to this dispute might have been clear based on their records, but for the fact that during their respective careers the cross country course differed in length: 2.5 miles in Thad's time (circa 1980), and 5000 meters in Thomas's time (circa 2019).

Using their best runs ever on their respective courses, we present evidence that Thad was the better runner in his day, the GOAT – the "Greater Of All Time"[1].

1 Data

1.1 Introduction and Runner Data

Both Thad and Thomas ran cross country for the same school (Bowling Green Senior High School, Bowling Green, Ohio). Thad ran many years before his son Thomas, however – 39 years before, to be exact. Like many father-son pairs before them, each man boldly asserts his preeminence to the other in their common sport; but how can one determine who was the better runner **as a senior** in high school? Which was the better runner in his glory days?

This is our quest; this is our charge[2].

The first thing we consider was how time has played a role in changing the sport, or how the sport was played in 1980 versus 2019. We asked the two runners if the school had changed in any appreciable way that would affect the cross country program, and neither identified any particularly strong issues. There were minor differences in class sizes (approximately 316 versus 270 students), but otherwise the programs seemed comparable. Aside some improvements in technology mentioned by Thad that have occurred since he was running (e.g. heart rate monitors, pace/mileage watches, portable rehab equipment, etc.) the competitors seem comfortable assuming that they ran under similar conditions in similar programs.

Each runner was asked for his career-best time in a cross country event.

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[1]Thad cannot be the "greatest", because this is merely a comparison between two people: we use a comparative adjective – "greater runner" – rather than a superlative adjective – "greatest runner" – because "greatest" would only be appropriate with three or more runners.

[2]This is Andy's Waterloo, because he's going to royally piss off either his brother or his nephew. What made him think that this was a good idea?

In the era in which Thad ran (circa 1980), cross country runners ran a 2.5 mile course. Thad's best time in his senior year (1980) was 13:30. In Thomas's era (circa 2019), runners ran a 5000 meter course ("5K"). Thomas's best time in his senior year (2019) was 17:15.

Although these were the best times for each, it may be that one of the two runners had far more favorable conditions than the other: weather, course lay-out and conditions, equipment, or other factors could have led to one runner having an extraordinarily good race. For that reason, students requested information on more races – e.g. the top three best times – from each runner, and so we obtained the second trove of data (Table 1).

Table 1: The three best times of each runner in their senior years.

Thad	Thomas
13:30	17:15
13:35	17:20
13:37	17:24

In particular, we note how little variation there is in these values. We deduce from this that each of these two is a consistent runner. One question that this additional information answers is the concern that one of these runners might have had an extraordinarily unusual and exceptional race – whereas the other might have been consistently good. It appears that both runners were able to perform at a high level on multiple occasions. The averages are 13:34 for Thad, and about 17:20 for Thomas, i.e. essentially the medians for both. Since there is so little difference from the best times, we decided to simply use the single best value in the analysis that follows.

In addition we obtained data on cross country champions in Ohio over the history of the school competitions (from 1931-2017), which was obtained at the website of OHSAA (the Ohio High School Athletic Association)[2, 3]; and we also obtained data on the 2019 regional and district races[5, 4].

1.2 Data Concerns

The data for our contestants were provided by the contestants themselves. Since they are on friendly terms, we trust that they have provided true and honest times.

In our analysis we assume that the two were running with comparable equipment: we ignore changes in technology, such as improvements in running shoes, shorter shorts, better socks, super-duper energy drinks, non-aerodynamic piercings, etc. We also assume that weather and course (and/or course condition) were neither an aid nor hindrance in the case of either runner.

In sum, we assume that the runners were racing under comparable conditions on comparable courses, and that the only difference was that one course was somewhat longer than the other. Our analysis compares their times as if they were running at the same time on the same day, same course, etc.

2 Methods

We now consider three different approaches to assessing the difference between these two runners. The first (pace) would allowed us to short-circuit the analysis, were Thomas to set a faster pace; the second introduces a comparison between each runner and the champions of his era, and we arrive at our first choice for GOAT ("Greater Of All Time"); finally we explore a formula developed in the literature and used widely on the web (Riegel's Formula), which allows us to estimate each runner's time at the other's distance – again leading to a definitive choice for GOAT.

2.1 Pace

5000 meters is approximately equal to 3.106856 miles.

If Thomas, who ran the longer race, had done so at a **faster** average pace than Thad, then he would clearly have claim to be the better runner; this is because we expect pace to **decrease** with course length (e.g. a sprinter, moving at an extraordinary pace, compared to a marathoner, moving at a far slower pace), and Thomas ran farther than Thad did.

However, Thad's best pace was actually a little faster than Thomas's (Thad's best one mile pace – 2.5 miles in 13:30 – was a 5:24 mile, whereas Thomas's pace was slightly more than a 5:33 mile – 5000 meters in 17:15); hence we must consider more than pace alone.

2.2 How Does Each Stack Up Against the Champions of His Day?

Obviously the different lengths of courses must come into play, and be dealt with appropriately. We introduce the data on past cross country champions at this point, as a way of "standardizing" the two course lengths.

We assume that the very best runners of each era knew how best to run their distance, their race; and we assume that the very best runners of one era are comparable to the very best runners of the other. Therefore, we compare Thomas and Thad against the champions of their times, considering each set of champions a common benchmark, controlling for the different distances.

There were three classes of school at the time of Thad, and three divisions at the time of Thomas. We took the 30 Ohio state champion runners' times (ten per class or division) closest by year to each runner to form a sample from the distribution of the best runners at that distance. (See the Appendix for this data, obtained from OHSAA[2].)

We checked the time samples for normality, and in one case the histogram did not appear normal, but rather appeared skewed right. Rather than consider times, we inverted the times and considered what were effectively speeds (data proportional to miles per minute). These data appeared more normally distributed, based on q-q plots. A Kolmogorov-Smirnov test on each failed to reject normality, and so we proceeded assuming normality in both cases[3].

From each sample we estimated the best-fitting normal distribution, with means and standard deviations, which are summarized (along with our runners' inverse times) in Table 2. The normal distribution analysis was performed in R, using commonly available routines such as fitdist.

From these hypothesized normal distributions underlying the speeds of the best runners

[3]Graphs of the inverted time distributions can be found in the Appendix.

Table 2: We used R and two different normal-fitting programs (including fitdist) to calculate the parameters of the best-fitting normal distribution; we tested normality using the Kolmogorov-Smirnov test, and could not reject normality. "Runner score" is simply obtained by computing $\frac{10}{time}$ (the choice of 10 was simply a convenience). One should not compare Thad's score with Thomas's score – each runner score is being compared to the scores of champions of his own era: this is effectively the same as saying that we should not compare Thad and Thomas's **times** (but that we **can** compare their times relative to the champions of their respective eras). The z-scores, however, should be compared – and it is on the basis of these that we declare Thad the GOAT.

Runner	Runner score	Mean μ	Standard deviation σ	z-score
Thomas	0.05797	0.06471	0.001284	-5.249
Thad	0.07407	0.08184	0.001813	-4.286

of each era, we computed z-scores for each runner,

$$z = \frac{x - \mu}{\sigma}$$

to see how far each runner fell from the mean. A z-score measures one's distance from the mean in terms of numbers of standard deviations (with a value of 0 indicating a value equal to the mean). The runner nearer the mean is declared the champion.

Each runner fell short of the mean, of course, being slower than the champions; hence, each runner has a negative z-score. That which is less negative is the winner.

Thomas's z-score:

$$Thomas = \frac{0.05797 - 0.06471}{.001284} = -5.249$$

Thad's z-score:

$$Thad = \frac{0.07407 - 0.08184}{.001813} = -4.286$$

This analysis declares Thad the champion, as his score is closest to 0 – less negative.

If we ignore the issue of normality, and simply compare Thad and Thomas to the respective mean of his group, we find that Thad's speed is 0.9052 of the average, whereas Thomas's speed is just 0.8958 of the average for his group of champions. This analysis would have Thad winning by a nose, and is similar in spirit to the normal distribution analysis which we just carried out.

In any event, we regret to inform Thomas that his father appears to have held a slight edge in cross country; however, we remind him that he may well have the opportunity to choose his father's nursing home.

2.3 Riegel's Formula

In 1977, Peter Riegel introduced a formula in an issue of Runner's World[8] which enables us to address this problem. Runner's World implemented this (and continues to feature this) formula on-line[1], i.e.

$$t_2 = t_1 \left(\frac{d_2}{d_1}\right)^{1.06}$$

where t_2 is your estimated time at distance d_2, given that we know your time t_1 at distance d_1.

Riegel went on to refine and expand his formula(s)[9], and in 1981 he published a detailed collection of results which he derived based on world record data, as well as by considering earlier work by authors such as Hill (who explored the physiological basis of athletic records[6]) and Keller (who explored a theory of competitive running[7]).

Riegel's equation is based on the apparent linearity of a plot of log(average speed) versus log(distance) of world class runners, and on the regression analysis he carried out based on that linearity.

Figure 1: Each of these lines is a best-fit line to a set of log-log transformed data. We focus on the top line, which represents data on world-class male runners at a wide variety of distances, and the best-fitting linear model which leads to Riegel's equation (Eq. (2)). This graphic is taken from Riegel's 1981 publication[9].

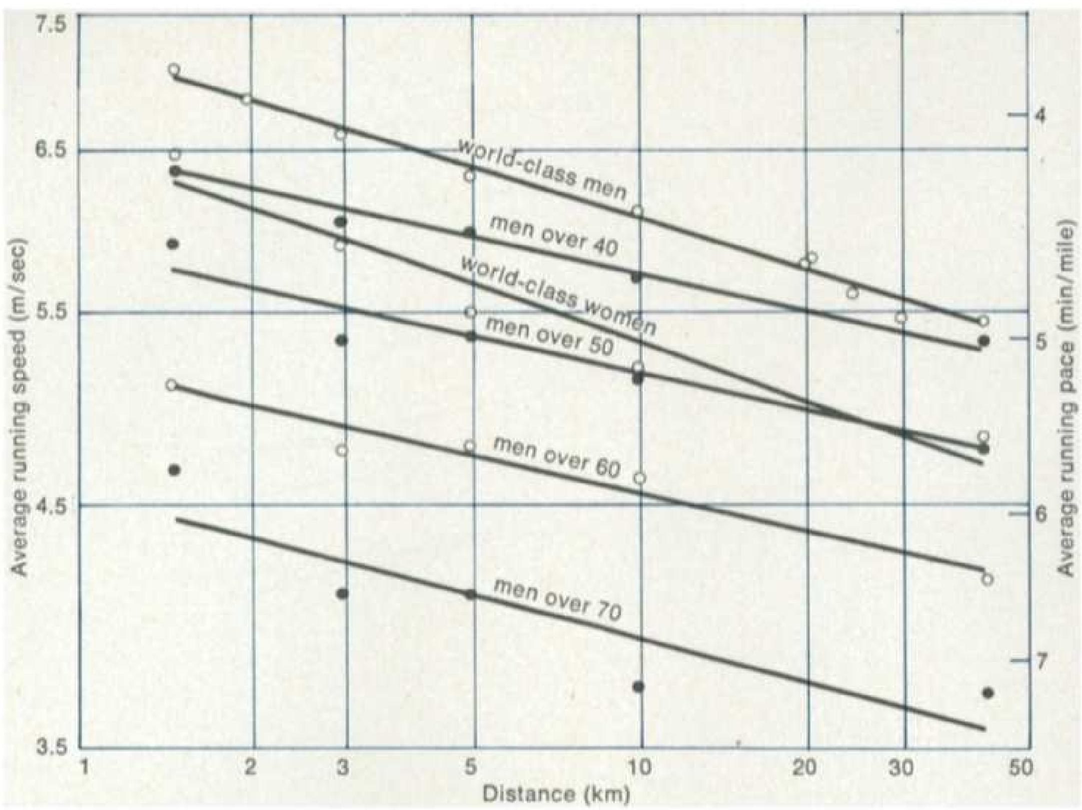


Figure 3. Runners provide the greatest amount of data for performance comparison. World records for various classes of runners highlight the parallelism between world-class men and women, and also among the men from 40 to 70 years of age. Parallel lines result from identical fatigue factors.

Using this data and log-log transformations, Riegel arrives at "power models" for times of elite athletes performing at different distances, of the form

$$t = a\, d^{\,b} \tag{1}$$

where t is time (in minutes), d is distance (km), and constants a and b (obtained by linear regression) differ for different age brackets and different sports. For the fittest male runners, the "world-class men", Riegel found the particular model

$$t = 2.299\ d^{1.07732} \tag{2}$$

One might be concerned about using Riegel's equation in the case of our (fit, yet slightly less than Olympian) runners Thad and Thomas, since it reflects the times of world-class runners. Riegel remarks in his paper that "[i]n an informal survey of distance runners in Ohio, I found that each tended to perform at some constant percentage of world-class speed over the entire competitive range of distances from mile to marathon. This suggests that the runner who is performing at, say, 70% of world-class speed at 10,000 m might expect to do the same at other distances."[9]

What this observation suggests is that Thad and Thomas will each be running at some (unknown) multiple α of the time in equation (2):

$$t = \alpha\ 2.299\ d^{1.07732}$$

where α is greater than 1 (since our subjects are presumably running a little slower – taking more time – than the Olympians). This constant does not present an obstacle, however, because we have one **known** time at a **known** distance. We proceed as follows: suppose one knows time t_1 at distance d_1, and wishes to know one's time t_2 at distance d_2. Then by Riegel's formula,

$$t_1 = \alpha\ 2.299\ d_1^{1.07732}$$

and

$$t_2 = \alpha\ 2.299\ d_2^{1.07732}$$

We could either solve for α using the known time and distance, or alternatively consider the **ratio**

$$\frac{t_2}{t_1} = \left(\frac{d_2}{d_1}\right)^{1.07732}$$

which eliminates α; we solve for t_2 as

$$t_2 = t_1 \left(\frac{d_2}{d_1}\right)^{1.07732}.$$

This is the derivation of the Runner's World equation[1] (although they chose to use 1.06 as the exponent, which was the earlier value Riegel gave in his article for Runner's World[8]). We expect that Riegel would side with 1.07732 in this case[4].

We compute the two runners' times on their competitors' distances:

$$Thomas_{2.5} = 17.25 \left(\frac{2.5}{3.106856}\right)^{1.07732} = 13.65$$

$$Thad_{3.1} = 13.50 \left(\frac{3.106856}{2.5}\right)^{1.07732} = 17.06$$

According to this analysis, we declare Thad the champion: Thomas would have run a 13.65 minute race at a distance of 2.5 miles, whereas Thad ran a 13.50 race; and Thad would have run a 17.06 minute race of 5000 meters, whereas Thomas ran it in 17.25 minutes.

Additional remarks:

[4] Riegel is deceased, unfortunately[10] – so we can't be sure.

a. We can compute the values of α directly; when we do so, we find that Thad's value of α is $\alpha = 1.311$, whereas Thomas's value is $\alpha = 1.325$. These values say that it effectively takes Thad and Thomas about a third longer than world-class runners to run the same race – with Thad just inching out his son.

b. If one uses the Runner's World exponent of $b = 1.06$ (rather than 1.07732), results only get better for Thad.

c. If one uses Thad's third best time, he still beats Thomas's best time, according to Riegel's formula.

3 Additional Considerations

Students in the modeling class brought up several other important perspectives and suggestions, which we feature here:

a. District[5] and regional[4] results for 2019 are also available, and these were used to enhance the data used in the analysis above (which featured results only to 2017[2]).

   The state champion in Thomas's division (I) in 2019 had a 5K time of 15:13 (Matt Duvall). Thomas's time of 17:15 was 2:02 behind the champion – 122 seconds. The state individual champion in Class III (Thad's division) in 1980 had a time of 11:36.7, whereas Thad ran a 13:30. So he was only 1:53.3 behind his champion – 113.3 seconds.

   One would expect the difference in times between winner and the other runners to spread out as the length of the race increased, however: for example, in the Riegel calculations above, the difference between predicted and known times for Thad and Thomas was .15 minutes for the 2.5 mile race, but .19 minutes for the 5K. So the question is whether Thad really has the advantage, coming out only 9.7 seconds ahead of Thomas in terms of raw time differences: 113.3 seconds for Thad, compared to 123 seconds for Thomas. The Riegel example suggests we might expect a ratio of

$$r = \frac{.19}{.15} = 1.267$$

   The actual ratio between our competitors and their champions was

$$R = \frac{122}{113.3} = 1.077$$

   This says that Thomas was actually closer in relative terms to his champion than Thad was to his, based on the Riegel expectation r.

   However one must ask whether it is fair to choose just one individual (the champion in a given year) against which to compare a runner: there is wide variation between champions from year to year. It makes more sense to compare Thad and Thomas against a distribution of champions, as we did above. In that case, if we compare each of Thad and Thomas against their "average champion" time, we find a ratio of

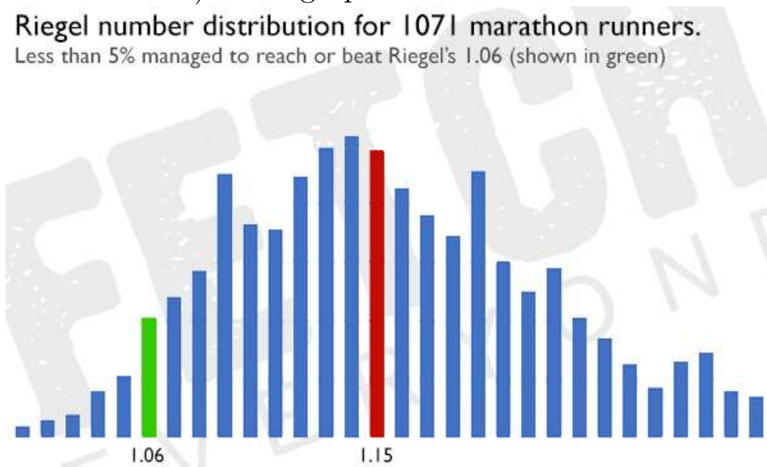
$$R = \frac{17.25 - 15.46}{13.5 - 12.23} = 1.41$$

   suggesting that Thad is really the closer to his champions than Thomas is to his.

b. One student used Riegel's formula in an interesting way.

There is some concern as to the validity of the exponent in Riegel's formula, at least for marathon runners[11]: as shown in figure 2, Riegel's exponent of 1.06 does a poor job of predicting marathoners' times. Ian Williams, author of the article, states that

Figure 2: A distribution of times, showing those who beat their estimated marathon times obtained using Riegel's 1.06 (based on half-marathon times) – those to the left of the orange bar – as well as those who those who beat their estimate using the revised exponent of 1.15 (those to the left of the red bar). This graphic was obtained from William's article[11].



"Riegel's classic formula uses R=1.06, which still provides excellent predictions for most distances even 40 years later - but out of 1,071 marathon runners in our Fetcheveryone sample (all of whom had completed at least five half and five full marathons), less than 5% managed a time that kept up with Riegel's predictions. The rest fell short."

The good news is that 1.06 provides a good prediction for shorter distances – at least according to Williams – and our two competitors are running much shorter distances. So Williams asserts that we can trust that 1.06 for our case, perhaps. However, what if that exponent were to change? As the exponent grows, it predicts that it will take a runner longer to complete a given course. What exponent would lead to Thad's time equaling Thomas's on the longer race? I.e.

$$Thad_{3.1} = 13.50\left(\frac{3.106856}{2.5}\right)^b = 17.25$$

We solve for b to obtain

$$b = \ln\left(\frac{17.25}{13.50}\right) / \ln\left(\frac{3.106856}{2.5}\right) = 1.128$$

None of the values of **b** obtained in Riegel's paper[9] for any class of runners equals 1.128. Only men's roller skating achieves a value of b this high – 1.13709; and so we are prepared to declare Thomas the better roller skater.

c. There is some concern that Thad never shared his lucky underwear with his son. Without knowing more about their underwear sizes, we are unable to determine whether failure to share was due to size incompatability, or whether Thad deliberately withheld from his son this important resource.

d. First, build a time machine; then send Thad back to 1980, and have him train on the 5K courses and beat Thomas's record. Better yet, have Thomas just travel back a couple of months, and beat Thad on a 2.5 mile race. We might even cut him some slack, and allow him to beat his dad with a few extra months of training under his belt.

   Let's skip the time machine, and if Thomas beats his dad's time on 2.5 miles of a cross country course that his father ran, within five months of the date of this report, we'll consider him the GOAT. Said one student, throwing out the gauntlet, "If Thomas is truly the better runner... even if he's out of practice he should be able to run a 2.5 mile better than Thad."

   This student continues: "If he refuses, he obviously concedes to being the slower runner (I have 2 brothers; this is logic)." Ball's in your court, Thomas....

4 Conclusions and Reflections

Comparing two players across the decades is never easy. In some cases, one can make a head-to-head comparison by letting two players compete directly – as occurred recently on Jeopardy's "Greatest of All Time" competition – Ken Jennings whomping on James Holzhauer from across the years.

In other cases, it's simply impossible – Babe Ruth versus Hank Aaron, Bill Russell versus LeBron James, Genghis Khan versus Attila the Hun, etc. And certainly in the case at hand here, of Thad versus Thomas, even Thad would agree that he is simply too old a fart to try to race his spry and nimble son Thomas: so a head-to-head race is right out.

Complicating the determination of the GOAT is the fact that the data we have on Thad concerns races of 2.5 miles, whereas data on Thomas concerns courses of length 5000 meters. So we face an "apples versus oranges" problem.

A simplistic rate calculation shows that Thad ran at a faster pace than Thomas; but this would be expected, given Thomas's longer race – so that is not enough to determine the better runner.

We used data obtained from OHSAA (the Ohio High School Athletic Association) to "standardize" Thad and Thomas's scores. The data set represented winning times for Ohio state champions in cross country races from across the Ohio history of the sport. We decided between Thad and Thomas by asking the question "How did each stack up against the champions of their time (and their course distance)?"

We selected thirty champions from three separate classes or divisions for each of our two contestants. The champions were those closest in year to Thad and Thomas, who had run races of the same length. We determined that speeds were relatively normally distributed, found best-fitting normals to each set of data, computed z-scores for Thad and Thomas's speeds, and determined based on this analysis that Thad inched out his son for GOAT. Furthermore, if we simply compute relative ratios of Thad and Thomas's speeds versus the means for each comparison group, we find that Thad again inches out Thomas.

Our discovery of Riegel's equation gave us a very good (and evidently fairly respected) method for evaluating each runner – "fairly respected", since Runner's World seems to think that it does a reasonable job of predicting times for races of different lengths. Riegel's equation suggests a fairly strong preference for Thad as GOAT.

Therefore, based on the relative merits of the two methods, we must declare this to be a tie[5].

5 Appendix

5.1 Data

Below is the data set used to compare Thad and Thomas (respectively) with champions of their era. These were stripped from a file obtained off of the OHSAA (the Ohio High School Athletic Association) website[2].

Thad:

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Year, Name, School, Minutes, Seconds

1977, I-John Locker, Twin Valley North, 12, 21.3
1977, T-Doug McDonald, Defiance Ayersville, 12, 34.9
1978, I-Dean Wood, Hicksville, 12, 40.1
1978, T-Earl Zilles, West Liberty-Salem, 12, 06.7
1979, I-Jeff Johnson, Yellow Springs, 12, 56.1
1979, T-Earl Zilles, West Liberty-Salem, 12, 24.4
1980, I-Scott Campbell, Ashtabula St. John, 12, 12.2
1980, T-Tony Thaman, Sidney Lehman, 12, 26.7
1981, I-Dave Rickerd, Patrick Henry, 12, 23
1981, T-Doug Lawrence, Lafayette Allen East, 12, 29.4

1977, I-Glen McCaslin, Cortland Lakeview, 12, 17.7
1977, T-Mike Maynard, Cincinnati Greenhills, 12, 09.9
1978, I-Chuck Bridgman, Dayton Chaminade-Julienne, 12, 05.3
1978, T-Joel Marchand, Navarre Fairless, 12, 17.8
1979, I-Joel Marchand, Navarre Fairless, 12, 11.9
1979, T-Mitch Bentley, McArthur Vinton County, 12, 32.2
1980, I-Mitch Bentley, McArthur Vinton County, 12, 03.4
1980, T-George Rodriguez, Cardinal Stritch, 12, 16.1
1981, I-Tom Franek, Chagrin Falls Kenston, 12, 14.7
1981, T-Ben Weeman, Orrville, 12, 12

1977, I-Gerald Vilt, Cleveland Rhodes, 12, 10.7
1977, T-Alan Scharsu, Youngstown Austintown-Fitch, 11, 46.8
1978, I-George Nicholas, Dayton Meadowdale, 11, 45.2
1978, T-John Zishka, Lancaster, 12, 14.3
1979, I-George Nicholas, Dayton Meadowdale, 12, 08.2
1979, T-John Zishka, Lancaster, 12, 12.9
1980, I-George Nicholas, Dayton Meadowdale, 11, 36.7
1980, T-Clark Haley, Lancaster, 11, 53.3
1981, I-Bob Mau, Rocky River, 12, 04.8
1981, T-Dean Monske, Toledo DeVilbiss, 11, 58
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Thomas:

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Year, Name, School, Minutes, Seconds

2008, Zach Wills, Mason, 15, 15.6
2009, Zach Wills, Mason, 15, 45.1
2010, Zach Wills, Mason, 15, 19.8
2011, Tsehaye Hiluf, Reynoldsburg, 15, 21.5
2012, Sam Wharton, Tipp City Tippecanoe, 15, 09.9
2013, Mark Hadley, Boardman, 15, 10.6
2014, Michael Vitucci, Cincinnati St. Xavier, 15, 34.5
2015, Andrew Jordan, Pataskala Watkins Memorial, 14, 54.0
2016, Dustin Horter, Liberty Township Lakota East, 15, 02.8
2017, Dustin Horter, Liberty Township Lakota East, 15, 03.4
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[5] "Which is the only way that I'll be able to live with the both of you," says brother/uncle Andy)

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2008, Jarrod Eick, Alliance Marlington, 15, 38.1
2009, Michael Brajdic, Bay Village Bay, 15, 49.8
2010, Michael Brajdic, Bay Village Bay, 15, 01.9
2011, Steven Weaver, Napoleon, 15, 25.1
2012, Mick Iacofano, Akron St. Vincent-St. Mary, 15, 32.9
2013, Matt Bromley, Thornville Sheridan, 16, 04.6
2014, Joseph Bistritz, Chagrin Falls, 15, 44.4
2015, Joseph Bistritz, Chagrin Falls, 15, 16.9
2016, Zach Kreft, Delaware Buckeye Valley, 15, 16.4
2017, Zach Kreft, Delaware Buckeye Valley, 15, 20.6

2008, Isaiah Bragg, Fairfield Cincinnati Christian, 15, 54.8
2009, Ryan Polman, Independence, 16, 01.6
2010, Colin Cotton, Cincinnati Summit Country Day, 15, 32.5
2011, Samuel Prakel, Versailles, 15, 19.3
2012, Samuel Prakel, Versailles, 15, 16.6
2013, Bobby Johnson, McDonald, 15, 45.4
2014, Tristan Dahmen, Cortland Maplewood, 15, 57.5
2015, Tristan Dahmen, Cortland Maplewood, 15, 18.8
2016, Chad Johnson, N. Robinson Colonel Crawford, 15, 18.4
2017, Chad Johnson, N. Robinson Colonel Crawford, 15, 33.3
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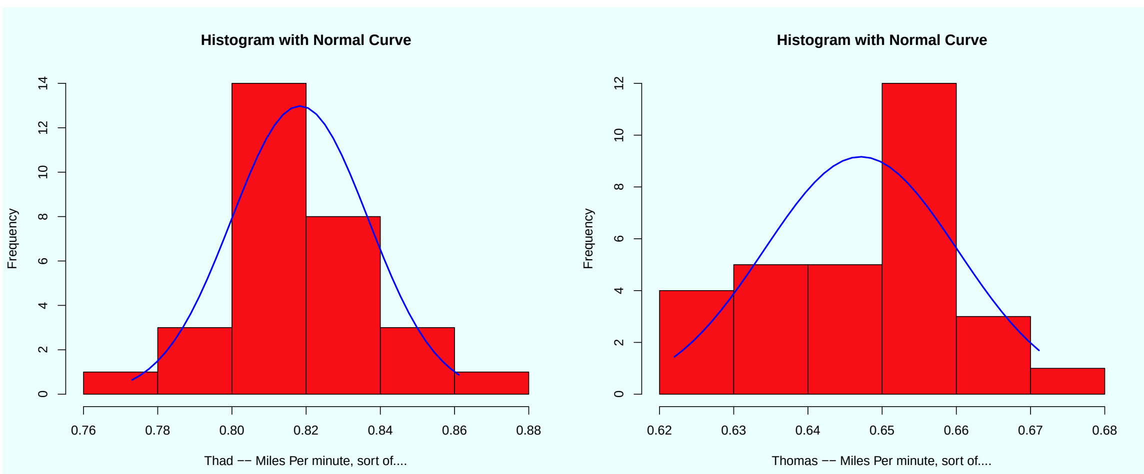
5.2 Histograms of Data



5.3 Riegel's Model

It might be better to write Riegel's model in a slightly different form. Units are the reason: equation (1) has units of time on the left, and so it must have units of time on the right:

$$t = a\ d^{\ b}$$

This means that the constant a must have units $(time/distance^b)$ – which is completely unintuitive. Riegel himself says that "The constant a is dependent on the units of measurement chosen and has no particular absolute significance, although it provides a measure of relative speed."

Let's re-write Riegel's model as

$$t = \frac{d}{V}\left(\frac{d}{D}\right)^{b-1} \tag{3}$$

introducing two new parameters, V and D, in place of the single parameter a. This may seem to be a loss, but each of these is **interpretable** – giving us information about the particular runners modeled – whereas a is not.

Interpretation: V **is the speed of the runner at distance** D**.**

Relating this to Riegel's equation (Eq. (1)), we have that

$$a = \frac{1}{V\ D^{b-1}}$$

or

$$D = \left(\frac{1}{a\ V}\right)^{\frac{1}{b-1}}$$

One is free to choose V to be any particular special speed, chosen however you'd like. For example, you might choose your own average speed in your favorite race. Once we know V, we can determine D from a. That is, this value of D will correspond to V: it is the distance which the world's best runners (rather than you) would run at that average speed V.

As an example, consider a 5K race, and assume that you run it in 15 minutes. Then one might wonder how far the world's best runners could run at that pace. We take

$$V = \frac{5000m}{15min} = \frac{1}{3}\frac{km}{min}$$

Then, using the coefficient of 2.299 from the Riegel equation (Eq. (2)), we obtain

$$D = \left(\frac{1}{2.299\ 1/3}\right)^{\frac{1}{1.07732-1}} = 31.25$$

That is, world-class runners could run 31.25K at your 5K pace. (How humiliating!) But if you'll consult Figure 1, you'll see that at a pace of $5.56\frac{m}{s}$ (V, expressed in units of $\frac{m}{s}$), the line passes through 31.25 km (approximately).

Finally, we note that you can think of this equation (3) in the following way: at distance D

$$t = \frac{D}{V}\left(\frac{D}{D}\right)^{b-1} = \frac{D}{V}$$

represents your time on a run of distance D at speed V. As you run d km (shorter or farther than the reference distance D), your time is scaled by the (dimensionless) factor

$$\left(\frac{d}{D}\right)^{b-1}$$

This factor will be greater than 1 when the distance is greater than D (times get longer, because your pace gets slower), or less than 1 when the distance is less than D – because you will be running more quickly than you would at distance D.

Hence if we built the equation off of Thomas's time, his equation would be

$$Thomas(d) = 17.25\left(\frac{d}{5}\right)^{1.07732}$$

References

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