Applied Math Modeling Final Exam (Spring 2020)

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). When you're done with the exam (you have two hours) please either email it back, or post it to your google drive. **Good luck, and best of luck moving forward.**

Problem 1. (10 pts) In class we demonstrated that the simple linear regression line must pass through the center of mass of the data, $(\overline{x}, \overline{y})$. Thus there is really only one "degree of freedom" for the line – the slope m. Use a little univariate calculus to **derive** the value of m, by minimizing the function

$$f(m) = \sum_{i=1}^{N} (y_i - \overline{y} - m(x_i - \overline{x}))^2$$

Problem 2. (10 pts) We like linear functions, operations, etc. in mathematics, because they tend to make our lives easy. Illustrate **graphically** how Newton's method is an example of using a linear method to solve a non-linear problem in the case of finding the root of $\ln(x)$ from a starting point of $x_0 = \frac{1}{2}$. I suggest a "plot window" of $[0, 2] \times [-2, 2]$, using the entire grid below.

No need to go beyond the first step of Newton's method. Describe as explicitly as you can

a. (3 pts) the non-linear problem,

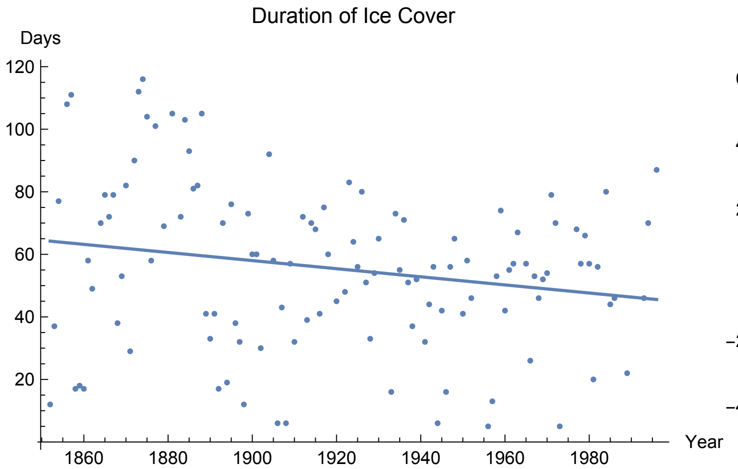
b. (4 pts) the associated linear problem, and

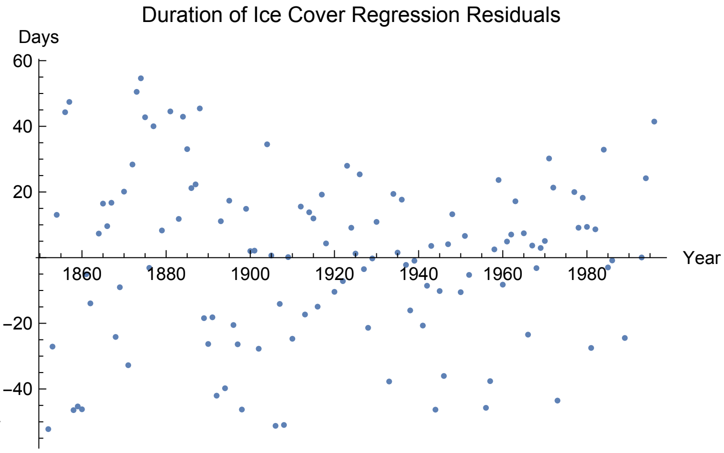
c. (3 pts) describe how we make use of the linear problem to solve the non-linear problem.

Problem 3. (10 pts) Ice on Grand Traverse Bay has been monitored for over 100 years. According to the 2012 EPA publication *Climate Indicators*, "The time that lakes stay frozen has generally decreased since the mid-1800s. For most of the lakes in this indicator, the duration of ice cover has decreased at an average rate of one to two days per decade."

a. Following are results of a regression analysis of the duration of ice cover for Grand Traverse Bay (Lake Michigan) through 1996. Critique this model, and interpret the coefficients in the context of lake ice cover for Grand Traverse Bay.

The data set ends in 1996, perhaps because (as one author notes) the bay "did not freeze up in five of the seven winters after the winter of 1993...."






```
dtrlm = LinearModelFit[data, {x}, x]
dtrlm["ParameterTable"]
dtrlm["AdjustedRSquared"]
dtrlm["ParameterConfidenceIntervals"]
dtrlm["ANOVATable"]
```

```
FittedModel[ 304.29 - 0.129625 x ]
```

	Estimate	Standard Error	t-Statistic	P-Value
1	304.29	111.985	2.71723	0.00759208
x	-0.129625	0.0583181	-2.22273	0.0281726

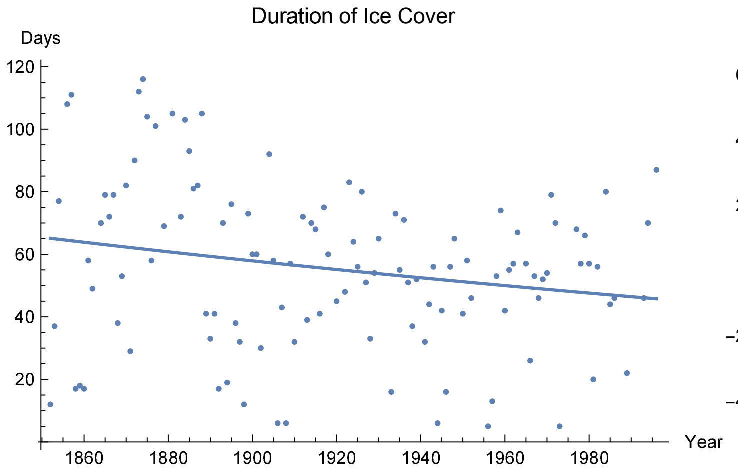
```
0.0325824
```

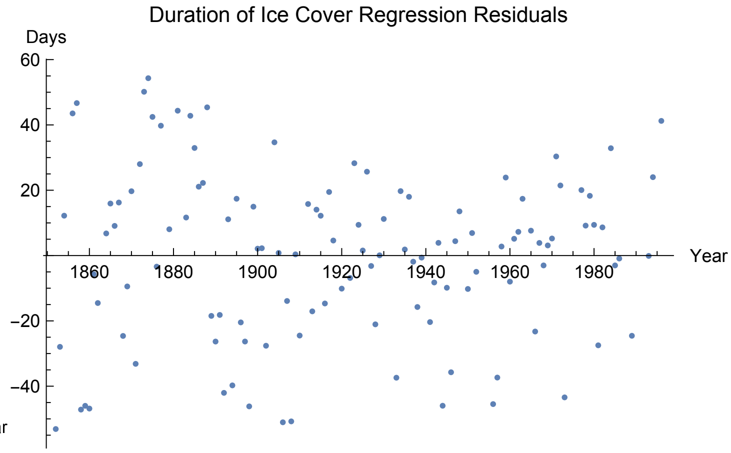
```
{{82.4888, 526.091}, {-0.245132, -0.014119}}
```

	DF	SS	MS	F-Statistic	P-Value
x	1	3257.73	3257.73	4.94053	0.0281726
Error	116	76 489.2	659.39		
Total	117	79 747.			

b. Following are results of a non-linear regression analysis of the duration of ice cover for Grand Traverse Bay (Lake Michigan) through 1996. Critique this model, and interpret the coefficients in the context of lake ice cover for Grand Traverse Bay.

Do you choose this model, $IC(t) = ae^{-b(t-1850)}$, over the linear model? Why or why not?






```
durnlm = NonlinearModelFit[data, a Exp[-b (x - 1850)], {{a, 800}, {b, 3 / 70}}, x]
durnlm["ParameterTable"]
durnlm["AdjustedRSquared"]
durnlm["ParameterConfidenceIntervals"]
durnlm["ANOVATable"]
```

FittedModel[$65.4289\, e^{-0.00244742\,(-1850+x)}$]

	Estimate	Standard Error	t-Statistic	P-Value
a	65.4289	5.13851	12.7331	9.0262×10^{-24}
b	0.00244742	0.00106768	2.29227	0.0236933

0.824418

{{55.2514, 75.6063}, {0.000332736, 0.0045621}}

	DF	SS	MS
Model	2	365 980.	182 990.
Error	116	76 348.5	658.177
Uncorrected Total	118	442 329.	
Corrected Total	117	79 747.	

Problem 4. (10 pts) Comment on the regression models featured in the following graphics, and sketch in a model you might prefer (if any – with justification).

a. This graph denotes the relationship between prices (relative to the United States, so that U.S.A=(1,1)) and economic output per capita (also relative to the US).

660 *Journal of Economic Literature, Vol. XXXIV (June 1996)*

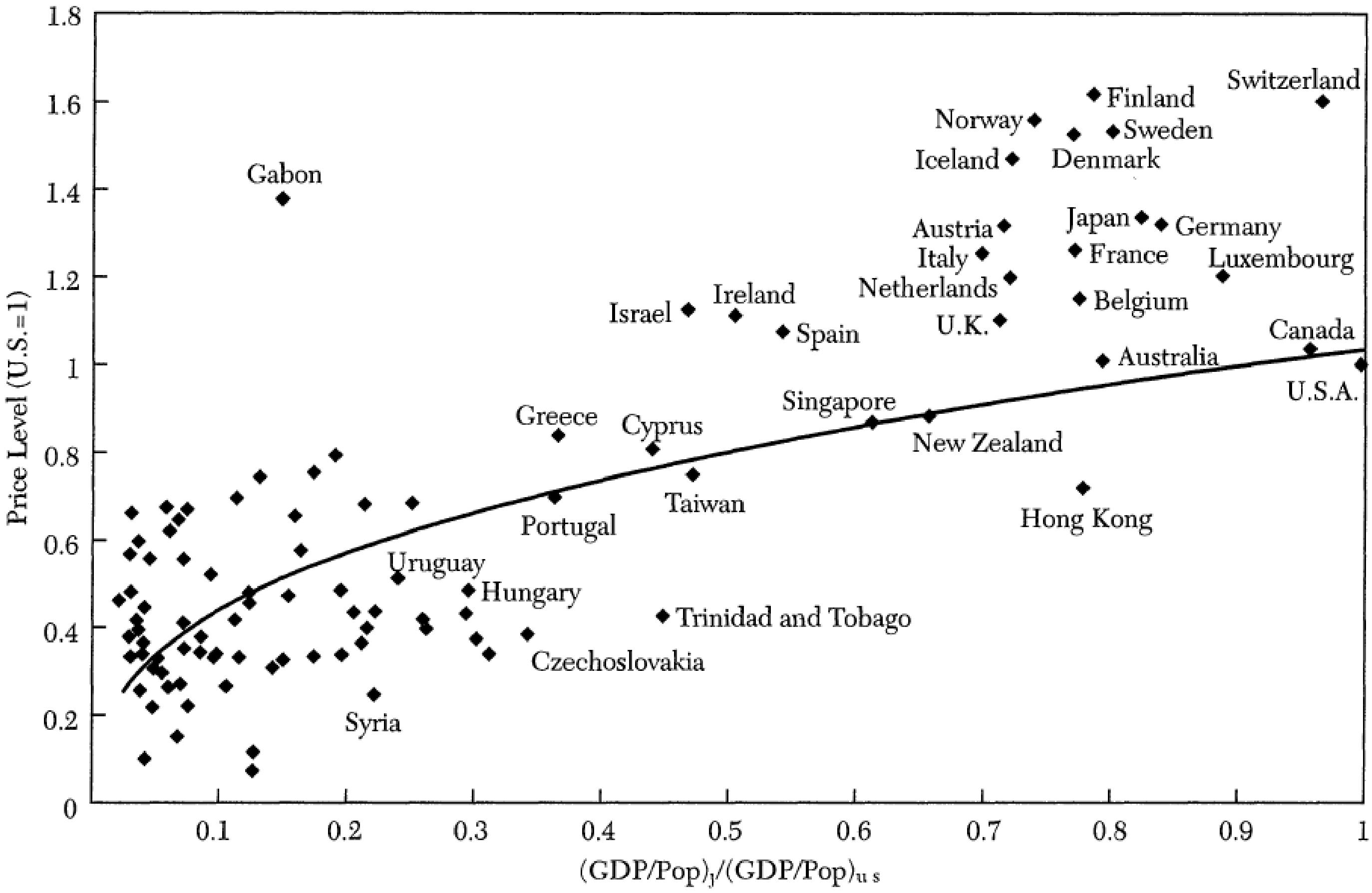
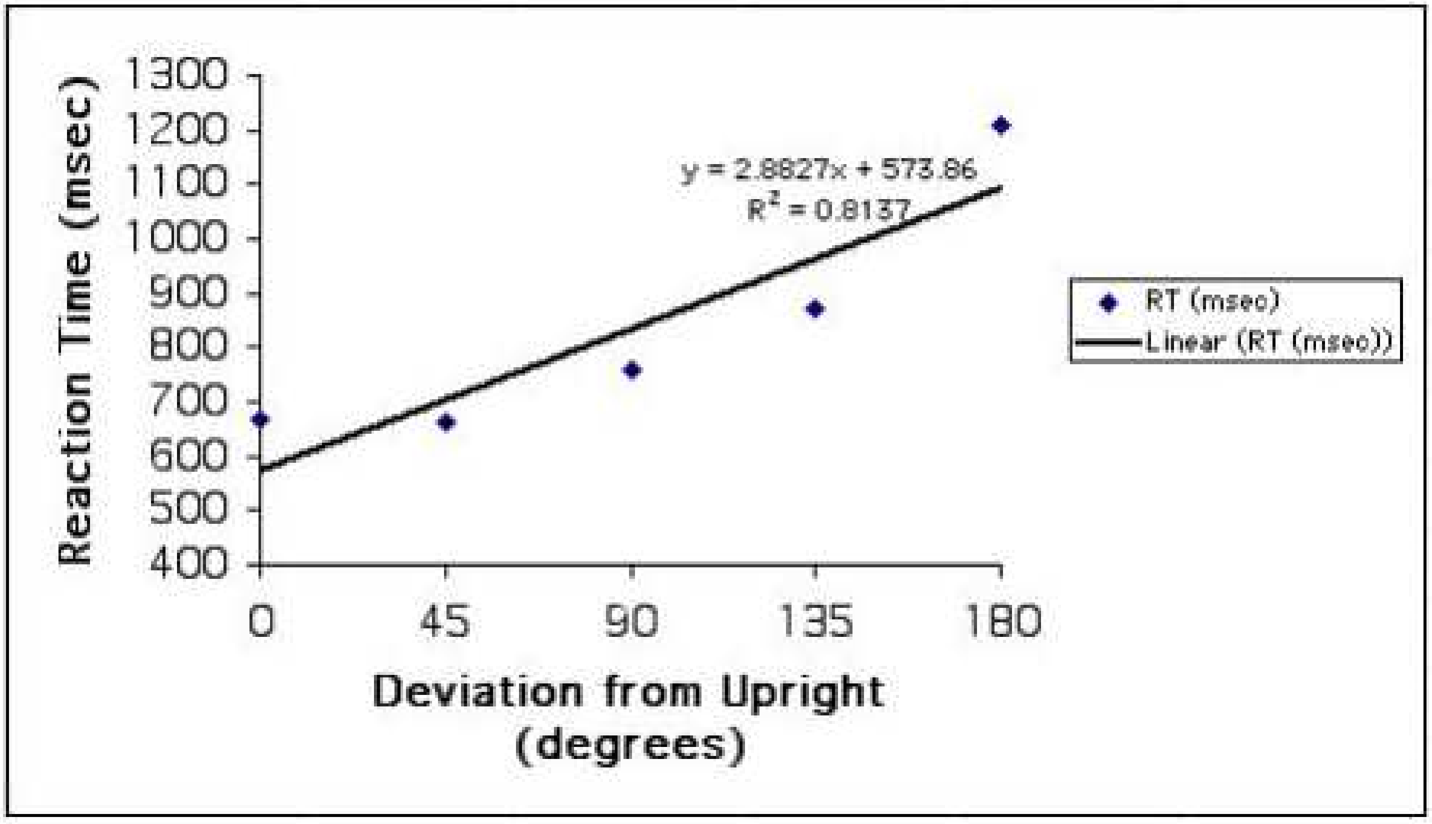


Figure 3. Price Level versus GDP per capita (U.S. = 1) 1990 $\log(P_j/P_{u.s.}) = \underset{(0.090)}{0.035} + \underset{(0.042)}{0.366} \log(Y_j/Y_{u.s.})$

Source: The Penn World Table, Aug. 1994

b. The following is from Psychology 403, Laboratory in Cognitive Psychology: "Doing a Linear Regression Analysis, Using Excel". Pretty good R^2...!



Problem 5. (10 pts) Regarding the Fletcher project, answer the following:

a. Fletcher suspected that weather would continue pretty much the way it had throughout his lifetime. What **specific details** makes us suspicious that such is not the case, based on our analysis of the Fletcher data?

b. Fletcher's data included ties for "Extreme Year" corresponding to an extreme temperature on a given day. They're a little inconvenient. How might we reasonably "handle them", if our objective is to determine the distribution of Extreme years over time (and, in particular, whether they are differentially distributed by climate change)?

c. A year contains 365.25 days or so (actually closer to 365.24219...). Our data spans 127 years or so. What are the consequences of assuming a climate model with a period of 365 days (or, worse yet, 366....)? Think about the climate for a particular day, like May 4th.

Problem 6. (10 pts) The logistic differential equation was used throughout our predator-prey unit, usually to model the prey.

a. Carefully graph **typical solutions** for the logistic DE

$$y' = sy(K - y)$$

including solutions with initial population $y_0 = y(0)$ below K as well as above K.

b. Explain the rationale for choosing this particular model for the prey. In particular, discuss the roles of s and K.