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$$(\forall x) [C(x) \rightarrow (\exists y) [W(x,y)]] \wedge$$

$$(\forall x)(\forall y) [W(x,y) \rightarrow S(x,y)] \wedge$$

$$C(m) \rightarrow (\exists y) [S(m,y)]$$

- | | | |
|-----|---|----------|
| 1. | $(\forall x) [C(x) \rightarrow (\exists y) [W(x,y)]]$ | hyp |
| 2. | $(\forall x)(\forall y) [W(x,y) \rightarrow S(x,y)]$ | hyp |
| 3. | $C(m)$ | hyp |
| 4. | $C(m) \rightarrow (\exists y) [W(m,y)]$ | 1, ui |
| 5. | $(\exists y) [W(m,y)]$ | 3, 4, mp |
| 6. | $W(m,y)$ | 5, ei |
| 7. | $(\forall y) [W(m,y) \rightarrow S(m,y)]$ | 2, ui |
| 8. | $W(m,y) \rightarrow S(m,y)$ | 7, ui |
| 9. | $S(m,y)$ | 6, 8, mp |
| 10. | $(\exists y) [S(m,y)]$ | 9, eg, |

$$\rightarrow (\forall x) [D(x) \rightarrow F'(x)]$$

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$$1. (\forall x) [F(x) \rightarrow (\exists y) [C(y) \wedge O(x,y)]] \wedge$$

$$2. (\forall x)(\forall y) [D(x) \wedge C(y) \rightarrow O(x,y)]$$

- | | | |
|-----|--|-------------------|
| 3. | $F(x)$ | hyp |
| 4. | $F(x) \rightarrow (\exists y) [C(y) \wedge O(x,y)]$ | 1, ui |
| 5. | $(\exists y) [C(y) \wedge O(x,y)]$ | 3, 4, mp |
| 6. | $C(y) \wedge O(x,y)$ | 5, ei |
| 7. | $(\forall y) [D(x) \wedge C(y) \rightarrow O(x,y)]'$ | 2, ui |
| 8. | $D(x) \wedge C(y) \rightarrow O(x,y)'$ | 7, ui |
| 9. | $O(x,y) \rightarrow (D(x) \wedge C(y))'$ | 8, cont. |
| 10. | $O(x,y)$ | 6, simp |
| 11. | $(D(x) \wedge C(y))'$ | 9, 10, mp |
| 12. | $D(x)' \vee C(y)'$ | 11, deM |
| 13. | $C(y)$ | 6, simp |
| 14. | $D(x)'$ | 12, 13, ds |
| 15. | $F(x) \rightarrow D(x)'$ | hyp
discharged |
| 16. | $D(x) \rightarrow F(x)'$ | 15, cont |
| 17. | $(\forall x) [D(x) \rightarrow F(x)']$ | 16, ug. |