$P(n)$:

To Prove: $F(n+4) = 3F(n+2) - F(n) \quad \forall n \geq 1.$

$F(1) = 1$
$F(2) = 1$
$F(3) = 2$
$F(4) = 3$
$F(5) = 5$

$5 = 3 \cdot 2 - 1$ ✓

Base cases! $n=1$ $P(1)$: $F(5) \stackrel{?}{=} 3F(3) - F(1)$

$n=2$ $P(2)$: $F(6) \stackrel{?}{=} 3F(4) - F(2)$
$8 = 3 \cdot 3 - 1$ ✓

We need multiple because we're going to be using the formula $F(n) = \underbrace{F(n-1) + F(n-2)}$

& we need to assert that the formula holds for the two preceding cases — which also means we'll be using the 2^{nd} principle!

Inductive step: Assume $P(r)$ holds for all $1 \leq r \leq k$, & consider the assertion

$P(k+1)$: $F((k+1)+4) = 3F((k+1)+2) - F((k+1))$

We'll start with the LHS (left-hand side):

$$F((k+1)+4) = F(k+5)$$
$$= F(k+4) + F(k+3)$$

$$= 3F(k+2) - F(k) + 3F(k+1) - F(k-1)$$

$$= 3\big(\underbrace{F(k+2) + F(k+1)}\big) - \big(\underbrace{F(k) + F(k-1)}\big)$$

$$= 3 \quad F(k+3) \quad - \quad F(k+1)$$

$$= \underbrace{3F((k+1)+2) - F((k+1))}_{\text{the RHS of } P(k+1)}$$

Therefore ✓

$\therefore P(n) \quad \forall n \in \mathbb{N}$ Natural numbers

by induction (2^{nd} principle)