Review Problems

§3.3 #9 Two recurrence relations:

$S(0) = a_0$

$P(0) = 1$ PI

$P(n) = c \cdot P(n-1)$

$S(n) = S(n-1) + a_n P(n)$

Bubblesort:

$B(1) = 0 \leftarrow a$

$g(n) = n-1$

$B(n) = \underset{c=1}{B(n-1)} + \overbrace{n-1}$

$$B(n) = 1^{n-1} \cdot 0 + \sum_{i=2}^{n} 1^{n-i} \cdot (i-1)$$

$$= 0 + \sum_{i=2}^{n} (i-1)$$

$$= \sum_{i=2}^{n} i - \sum_{i=2}^{n} 1$$

$$= 1 - 1 + \sum_{i=2}^{n} i - (n-1)$$

$$= -1 + \sum^{n} i - (n-1)$$

$i=1$

$$= \cancel{-1} + \frac{n(n+1)}{2} - n \cancel{+1}$$

$$= \frac{n^2+n}{2} - \frac{2n}{2} = \frac{n^2-n}{2}$$

$$= \frac{n(n-1)}{2}$$ quadratic

Mergesort ~ $n \log_2 n$

$1 + 2 + 3 + \dots + n$

$n + n-1 + \dots \quad - \quad + 1$

§3.2

#25

$P(1) = 1$

$P(n) = P(n-1) + 3n - 2$

$P(2) = 1 + (6-2) = 5$ ✓

§3.1 Define binary strings w/ even # of 1s.

Base cases: $\{\lambda, 0\}$ $\quad 0 = \lambda 0$

Inductive step: If x is a string w/ even # of 1s, then so are $\begin{cases} x0 \wedge 0x \\ \wedge 1x1 \end{cases}$

00 $\quad$ 0101

11 $\quad$ 1111

11011 $\quad$ 110011

$0 \to 00$

$\to 1001$

$\to 110011$

§2.2 #44

$P(n): 1 + \frac{1}{4} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n}$ (for $n \geq 2$)

Check the base case

$P(2): 1 + \frac{1}{2^2} < 2 - \frac{1}{2}$

$1 + \frac{1}{4} = \frac{5}{4} < \frac{6}{4} = \frac{3}{2} = 2 - \frac{1}{2}$ ✓

Assume $P(k)$: $1+\frac{1}{2^2}+\dots+\frac{1}{k^2}<2-\frac{1}{k}$. Show

$P(k+1)$: $1+\frac{1}{2^2}+\dots+\frac{1}{k^2}+\frac{1}{(k+1)^2}<2-\frac{1}{k+1}$.

Consider the Left Hand Side:

$$\underbrace{1+\frac{1}{2^2}+\dots+\frac{1}{k^2}}_{\substack{<2-1/k \\ \text{by } P(k)}}+\frac{1}{(k+1)^2}<2-\frac{1}{k}+\frac{1}{(k+1)^2}$$

$$=2-\left(\frac{1}{k}-\frac{1}{(k+1)^2}\right)$$

$$=2-\left(\frac{(k+1)^2-k}{k(k+1)^2}\right)$$

$$=2-\left(\frac{k^2+2k+1-k}{k(k+1)^2}\right)$$

$$=2-\left(\frac{k^2+k+1}{k(k+1)^2}\right)$$

$$<2-\left(\frac{k^2+k}{k(k+1)^2}\right)$$

$$=2-\frac{k(k+1)}{k(k+1)^2}$$

$$=\boxed{2-\frac{1}{k+1}}$$

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§1.4 #34

1.

2.

3.

4. $A(x) \wedge N(x)'$ 1, ei FIRST!

5. $G(x) \to N(x)$ 2, ui

6. $G(x) \vee C(x)$ 3, ui

7. $N(x)'$ 4, simp

8. $G(x)'$ 5, 7, mt

9. $C(x)$ 6, 8, ds

10. $A(x)$ 4 simp

11. $A(x) \wedge C(x)$ 9, 10 conj

12. $(\exists x)[A(x) \wedge C(x)]$ 11, eg.

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§1.3 #30

a. Some non-student eats pizza

b. Some student doesn't eat pizza

c. All students eat something other than pizza...